IJFANS INTERNATIONAL JOURNAL OF FOOD AND NUTRITIONAL SCIENCES
ISSN PRINT 2319 1775 Online 2320 7876
Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 2, Apr 2022

EXPLORING SOME IMPORTANT PROPERTIES OF MERSENNE NUMBERS

Sri. Lakshminarayana K. S.
Assistant Professor, Department of Mathematics Sri Bhuvanendra College - Karkala
Mail: lakshminarayanaks.666@gmail.com

ABSTRACT

This paper gives some important properties of special type of prime numbers called Mersenne primes after the French Mathematician Martin Mersenne. These are very rare and largest also. The largest Mersenne prime number has around 24 million digits. These are very useful in cryptography, computational applications etc

KEY WORDS: Prime numbers, Composite numbers, perfect numbers, Mersenne numbers.

INTRODUCTION:

The theory of numbers mainly delas with the properties of positive integers. These numbers together with zero and negative integers form the set of integers. The integers >1 are either prime or composite. There are infinitely many prime numbers, Every positive divisor of an integer > 1 is always prime. A special type of positive integers of the form $M_n= 2^n -1$, n>1 are called Mersenne numbers. Prime numbers are one whose positive divisors are 1 and itself. The development of this number theory was a very major advancement in the number theory. There are many mathematicians contributed to the number theory. One such mathematician was the French man, Martin Mersenne (1588- 1648). His contribution to number theory was in the field of prime numbers. Mersenne primes are special type of prime numbers. It was developed in the search of perfect numbers. A number is perfect if it the sum of its proper divisors. Example 6= 1+2+3. 6 is a perfect number. The prime numbers are considered as the building block of the number theory. If Mer senne numbers are prime, thery are called Mer senne primes.

In the seventieth century, when Mersenne made his famous conjecture, manually one has to calculate the numbers. Now the largest known Mersenne prime is $2^{43112609}$ -1 and research is going on to extend this.

DEFINITIONS AND PROPERTIES OF MERSENNE NUMBERS AND PRIMES

The numbers $M_n= 2^n -1$, n>1 are called Mersenne numbers. A Mersenne number which is prime is called a Mersenne prime.

Few Examples of Mersenne numbers are M_1=1, M_2= 3, M_3 = 7, …. , M_7 = 127 etc.

Few Examples of Mersenne primes are M_2= 3, M_3 = 7, M_5 = 31, M_7 = 127 etc .

M_4=15 is not a Mersenne prime.

Properties:

1. $2^{k-1} M_k$ is always a perfect number.

Example : For k=2 , $2^{k-1} M_k$ = 6 = 1+2+3, sum of proper divisors of 6 , which is a perfect number .

2. There is a one to one correspondence between perfect numbers and Mersenne primes
3. If p is prime and p> 2 then any prime divisors of M_p must be of the form 2kp +1 for k ∈N.
4. If p and q=2p+1 are both odd primes and p≡ 3(mod 4) then q/ M_p.
5. If q= 2n+1 is prime then q/ M_n provided q≡ ±1(mod 8)

Result: For p> 2 every prime divisor of Mp is of the form 8k±1.

Proof: Let Mp be an odd prime. Let q be any prime with q/ Mp.

Then q is odd and let q= 2m+1

Let a= .$2^{\frac{p+1}{2}}$

Then $a^2 = 2^{p+1} \implies$

$a^2 - 2 = 2^{p+1} - 2 = 2(2^p) - 2 = 2(2^p - 1) = 2$ Mp = 2(q.k1) as q/ Mp.

$\therefore$ a^2 -2 = 2(qk1) $\Longrightarrow$ q/(a^2 - 2) $\Longrightarrow$ $(a^2 - 2) \equiv 0$(mod q)
$\Longrightarrow$ $a^2 \equiv 2$ (mod q)
$\Longrightarrow$ $a^{2m} \equiv 2^m$(mod q) (As q= 2m+1 $\Longrightarrow$ 2m = q-1)
$\Longrightarrow$ $a^{q-1} \equiv 2^m$(mod q) (1)
As q is odd we have (a, q) =1 $\therefore$ By Fermat theorem $a^{q-1} \equiv 1$(mod q) (2)
From (1) and (2) we have $2^m \equiv 1$(mod q) $\Longrightarrow$ q/$(2^m - 1)$ $\Longrightarrow$ q/ Mm
q$\equiv \pm 1$(mod 8) $\Longrightarrow$
$\therefore$ Every divisor of Mp is of the form 8k ±1
Result : M_{29} is composite.
Proof: Let p be a prime with p/ M_{23} Then p= 2 (29k)+1
$\Longrightarrow$ p= 58k+1 for k ϵN.
When k = 1, we have p= 59, k=2, p= 117, k=3 p=175 , k- 4 , p=233.
Here 59 and 233 are primes. Also 233/ M_{29} .
$\therefore$ M_{29} is composite.
There are many properties which are frequently used in the other branches of science also

APPLICATIONS

It has wide applications in cryptography especially in public key algorithms and secure data transmissions, Computational number theory, In distributed computing, a process where several computers are connected through the network and cooperate in solving a problem and used in the study of perfect numbers etc.

CONCLUSION

This paper has explored the fascinating world of Mersenne primes, it's some properties, applications,. Through our examination, we have seen the significant impact of Mersenne primes on various fields, including number theory, cryptography, and computer science.

As we continue to explore the vast landscape of prime numbers, Mersenne primes remain a captivating and essential area of study. Their unique properties and applications make them an invaluable tool for advancing mathematical knowledge and addressing real-world challenges.

FUTURE RESEARCH DIRECTIONS MAY INCLUDE THE FOLLOWING:

a) Investigating the distribution and density of Mersenne primes
b) Exploring new applications in cryptography and computer science
c) Developing more efficient algorithms for discovering and verifying Mersenne primes

Ultimately, the study of Mersenne primes not only enriches our understanding of mathematics but also contributes to the development of secure and efficient technologies that impact our daily lives.

REFERENCES:

1. Buske, D, & Keith, S. (2000). GIMPS finds another prime. Math Horizons, 7(4), 19- 21.
2. Mersenne Prime # of Digits Calculator. (2008). In Gilchrist online. Retrieved from http://gilchrist.ca/jeff/MprimeDigits/ index.html
3. O'Connor, J. J., Robertson, E. F. (2009). Perfect Numbers. Retrieved from http://www-history.mcs.st-and.ac.uk/ Hist Topics/Perfect_numbers.html
4. Prime Number. In Nombre Premier online. Retrieved from http://fr.academic.ru/dic. nsf/frwiki/1240555