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SET-DOMINATION NUMBER OF SUB-DIVISION GRAPH

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ABSTRACT

In this paper, we initiate the study of Set-Domination in the Sub-Division graph of a graph G. The Sub-division graph $S_1(G)$ of a graph G is a graph obtained from G by inserting a vertex of degree two for each edge of G.Let $S_1(G) = (V, E)$be a Sub-division graph. A Set $D \subset V[S_1(G)]$ is a Set-dominating set (Sd-set) of $S_1(G)$ if, for every subset $T \subset V - D$, there exists a non-empty subset $S \subset D$ such that the graph $< S \cup T >$ is connected. The Set-Domination number $\gamma_{sd}[S_1(G)]$ of G is the minimum cardinality of an SD-set. Here we find Set Domination number of Subdivision graphs of some standard graphs and specifically subdivision graph of a non-trivial tree and create comparative inequalities between $\gamma_{sd}[S_1(G)]$ and other domination parameter of the graph G.

Keywords: Domination, Sub-division graph, Set-domination, Weak domination, Co-total domination, Non-split domination.

Subject Classification Number: AMS 05C69, 05C70, 05C20.

1. INTRODUCTION AND MOTIVATION

We consider here only simple, connected ,undirected graphs, free of loops and multiple edges. We write vertex set of a graph G as $V(G)$ with $|V(G)| = p$ and edge set as $E(G)$ with $|E(G)| = q$. Let $G = (V, E)$ be a connected graph a set $D \subset V$ is a dominating set of G,if every vertex in $V - D$ is adjacent to some vertex in D.The domination number $\gamma(G)$ of G is the minimum cardinality overall dominating sets of G. A set $D \subset V$ is a Set-dominating set (sd-set) of G, if for every set $T \subset V - D$, there exists a non-empty subset $S \subset D$ such that subgraph $< S \cup T >$ is connected. The Set-Domination $\gamma_{sd}(G)$ is the minimum cardinality taken over all the sd-sets of G. Sampath Kumar and E. Pushpa Latha proposed the concept of Set-domination in [23].They established certain general graphs and a set-domination number of standard graphs.And in fact, this inspired us to determine the set-domination number of some graph valued funcations. Many significant inequalities for the Set-domination number are demonstrated in [1], [2], and [3], which shows weak domination of the Semitotal block graph.The domination in Sub-division graphs is examined in [14] as well. Many other graph valued functions in graph theory were studied, for example, in [4, 8]. All of this gave us the incentive to determine the Set-Domination number of subdivision graphs.We were able to accomplish this quite successfully, since we discovered precise formulas for the set-domination of standard graphs and formulas that contrast the set-domination number of subdivision graphs of tree graphs and general graphs G with other parameters related to the original graph G.

We discuss the notations and preliminary definitions in section 2, followed by results established by us in section 3. Ultimately Section 4 provides the possible applications of Set-domination number in Subdivision graphs followed by Section 5 with conclusions.

2. NOTATIONS AND PREREQUISITES

We use [14] for terminology and notations, which are not defined here.The degree of a vertex $v \in V(G)$ is $deg_G(v) = |N_G(v)|$.The minimum and maximum degrees in graph G are denoted by $\delta(G)$ and $\Delta(G)$,respectively. We denote P_p a Path on p vertices. The distance $d(u, v)$ between two vertices u and v in a graph G is the minimum number of edges of a path from u to v.We denote C_p a cycle graph with p vertices. Star graph as K_p, Tree graph as T_p, Fan graph as F_p and W_p as the Wheel graph.

Definition 2.1 Dominating set first introduced by Oystein Ore [22] and except for Berge [9] and Harary [12], this term has been popularized in subsequent literature by many authors like Cockayne ,Hedetniemi and Slater.A set $D \subseteq V$ is a dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The Dominating number $\gamma(G)$ of G is the minimum cardinality of a Dominating set D. See [12].

Definition 2.2 A dominating set D is called the total dominating set if the subgraph induced by D has no isolated vertices.The minimal cardinality of D is called the total domination number of G and is denoted by $\gamma_t(G)$. Refer [15].

Definition 2.3 The concept of connected domination in graphs was introduced by Sampathkumar and Walikar [25] in 1979.A dominating set $D \subseteq V(G)$ is a connected dominating set if the induced subgraph $< D >$ is a connected subgraph of G. The connected domination number $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set of G. See [11].

Definition 2.4 A dominating set of D of a graph G is a cototal dominating set if induced subgraph $< V(G) - D >$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ is the minimum cardinality of a cototal dominating set of G. [20]

Definition 2.5 D is called weak domination set if $deg(u) \leq deg(v)$ for every pair of $(u, v) \in V(G) - D$. The minimum cardinality of a weak domination set D is called weak domination number and denoted by $\gamma_w(G)$ [2].

Definition 2.6 In a graph G, a vertex dominates itself and its neighbours. A subset $S \subseteq V(G)$ is a double dominating set of G if S dominates every vertex of G at least twice. The minimum cardinality of a double dominating set of G is the double domination number γ×2(G). See [13].

Definition 2.7 The vertices and edges of a graph G are called the elements of G. A set X of elements in G is an entire dominating set if every element not in X is either adjacent or incident

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to at least one element in X. The entire domination number $\gamma_{en}(G)$ of G is the minimum cardinality of an entire dominating set in G. This concept was introduced by Kulli in [21].

Definition 2.8 A dominating set $D \subseteq V$ of a graph G is a split (non-split) dominating set if the induced subgraph is disconnected (connected). The split (non-split) domination number $\gamma_s(G), [\gamma_{ns}(G)]$ is the minimum cardinality of a split (non-split) dominating set [19].

3. RESULTS

Following are the Set-domination number of standard graphs in terms of other graph invariants and other domination parameters which we merely state.

Theorem 3.1

a. For any path P_p graph with $p \geq 3$

$$\gamma_{sd}[s_1(P_p)] = \left\lceil \frac{p(P_p) + q(P_p)}{3} \right\rceil$$

b. For any Star $K_{1,p}$ with $p \geq 4$,

i. $\gamma_{sd}[s_1(K_{1,p})] = p - 1.$

ii. $\gamma_{sd}[s_1(K_{1,p})] = \gamma_{cot}(K_{1,p}).$

c. For any Cycle C_p with $p \geq 3$, Domination chain follows

$$\gamma_{sd}[s_1(C_p)] \geq \gamma_t(C_p) \geq \gamma_{ss}(C_p) \geq \gamma_{dd}(C_p) \geq \gamma_w(C_p) \geq \gamma(C_p) \geq \gamma_s(C_p).$$

d. For any Fan F_p graph with $p \geq 5$ vertices,

i. $\gamma_{cot}(F_p) + 1 = \gamma_{sd}[S_1(F_p)]$

ii. $\gamma_{en}(F_p) = \gamma_{sd}[S_1(F_p)]$

iii. $\alpha_1(F_p) = \gamma_{sd}[S_1(F_p)]$

The following theorem relates set-domination number of Sub-division graph and number of edges of a tree graph.

Theorem 3.2: For any connected tree graph $T_p, \gamma_{sd}[S_1[(T_p)] \leq q$.

Proof: Suppose D_1 is a set-dominating set of $S_1(T_p)$.Then by definition of set-domination $|V[S_1(T_p)]| \geq 2$.Further by definition of $S_1(T_p), \gamma_{sd}[S_1(T_p)] \leq q$. Clearly it follows that $\gamma_{sd}[S_1(T_p)] \leq q$.

The following theorem relates set-domination number of Sub-division graph and maximum vertex degree of a tree graph.

Theorem 3.3: For a non-trivial tree graph T_p,

$$\gamma_{sd}[S_1(T_p)] \geq p - \Delta(T_p)$$

Proof: Suppose G is a tree, then $A(T_p) = \{e_1, e_2, e_3, e_4, \dots \dots . e_n\}$ be the set of all end edges and $A'(T_p) = \{e_1, e_2, e_3, \dots . . e_i\} \forall e_i \in A(T_p)$ be the set of all non-end edges and $V(G) = \{v_1, v_2, v_{3,} \dots . . v_n\}$, $1 \leq i \leq n$. Let $D' = \{v_1, v_2, v_3, \dots . . v_n\}$ be the minimal dominating set of $S_1(T_p)$.

Further let $T \subset V[S_1(T_p)] - D'$ and $S \subset D'$ where $S \neq \emptyset$ and $< S \cup T >$ is connected. Then $|D'| = \gamma_{sd}[S_1(T_p)]$.Since $p = q + 1$ and by the above observation, it is clear that $\gamma_{sd}[S_1(T_p)] < p$ so that $|D'| \leq |V(T_p)| - \Delta(T_p)$ and hence $\gamma_{sd}[S_1(T_p)] \leq p - \Delta(T_p)$.

Theorem 3.4: For any non-trivial tree $T_p, p \geq 3$,

$$\gamma_{sd}[S_1(T_p)] \geq \gamma_{en}(T_p) + 1$$

Proof: We will prove the inequality by contradiction.

Assume that $\gamma_{sd}[S_1\,(T_p)] < \gamma_{en}\,(T_p) + 1$ for some non-trivial tree T_p, where $p \geq 3$.Let D be a minimum dominating set of $S_1(T_p)$ with $|D| = \gamma_{sd}[S_1\,(T_p)]$
Now let's consider the tree T_p. By definition, a dominating set in T_p is a set of vertices such that every vertex in the tree is either in the set or adjacent to a vertex in the set.
Let I be a set of vertices in T_p such that no vertex in I is adjacent to any other vertex in I, and $|I| = \gamma_{en}\,(T_p)$. In other words, I is an independent dominating set of T_p.
Now, we construct a new set D' as follows:Start with D.

- For each vertex v in I, add a vertex adjacent to v to D'.

Since D is a minimum dominating set of $S_1\,(T_p)$, we can conclude that D' is a dominating set of $S_1\,(T_p)$. Moreover, $|D'| = |D| + |I|$.
However, we have $|D'| = \gamma_{sd}[S_1(T_p)] + \gamma_{en}(T_p) > \gamma_{sd}[S_1(T_p)] + \gamma_{en}(T_p) + 1$, which contradicts our assumption.
Thus $\gamma_{sd}[S_1(T_p)] \geq \gamma_{en}(T_p) + 1$ for any non-trivial tree T_p, where $p \geq 3$.
Theorem 3.5: For any non-trivial tree T_p,

$$\gamma_{sd}[S_1(T_p)] \leq p - \delta(T_p)$$

Proof: Let $D = \{v_1, v_{2,} v_3, \dots \dots . v_m\}$ be the minimal dominating set in T_p and $V' = v - D$.Now, there exists atleast one vertex v of minimum degree $\delta(T_p) \epsilon V'$ in T_p.

Let $D_1 = \{v_1, v_2, v_3, \dots . . v_n\}$ be a set-dominating set of $S_1(T_p)$ such that $|D_1| = \gamma_{sd}[S_1(T_p)]$. Then $|D_1| \leq |V(T_p)| - \delta(T_p)$ which gives $\gamma_{sd}[S_1(T_p)] \leq p - \delta(T_p)$.

4. APPLICATIONS

Set-domination in sub-division graphs can be used to analyze the vulnerability of transportation systems, such as road networks or railway networks. By identifying dominating sets in sub-division graphs, we can determine the critical nodes that, if compromised or disrupted, would have a significant impact on the entire system. This analysis aids in identifying weak points and devising strategies to strengthen the system's resilience.

5. CONCLUSION

In conclusion, vulnerability analysis using set-domination in sub-division graphs is a valuable approach for assessing the vulnerability of transportation systems. By identifying critical nodes, this analysis helps in understanding weak points and devising strategies to strengthen the system's resilience. It has practical applications in transportation planning, management and identifying critical infrastructures.

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