

ON RADIO ANALYTIC MEAN Dd - DISTANCE NUMBER OF SOME MODERN GRAPHS

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ABSTRACT

A Radio analytic mean Dd -distance labeling of a connect graph G is an injective function f from the vertex set $V(G)$ to the \mathbb{N} such that for two distinct vertices u and v of G , $D^{Dd}(u, v) + \left\lceil \frac{f(u)^2 - f(v)^2}{2} \right\rceil \geq 1 + diam^{Dd}(G)$, where $D^{Dd}(u, v) = D(u, v) + deg(u) + deg(v)$, $D^{Dd}(u, v)$ denotes the Dd -distance between u and v $diam^{Dd}(G)$ denotes the Dd -diameter of G . The radio analytic mean Dd -distance number of f , $ramn^{Dd}(f)$ is the maximum label assigned to any vertex of G . The radio analytic mean Dd -distance number of f , $ramn^{Dd}(G)$ is the minimum value of G , $ramn^{Dd}(G)$ is the minimum value of $ramn^{Dd}(f)$ taken over all radio analytic mean Dd -distance labeling f of G . In this paper we find the radio analytic mean Dd -distance number of some Modern graphs.

KEYWORDS: Dd -distance, radio analytic mean Dd -distance, radio analytic mean Dd -distance number.

1. INTRODUCTION

A graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. The $O(G)$ and size of G are denotes by p and q respectively.

The Dd -distance concept was introduced by A. Anto Kinsley and P. Siva Ananthi. . We introduce the concept of radio analytic mean Dd – distance number of some basic graphs. For a connected graph G , the Dd -length of a connected $u - v$ path is defined as $D^{Dd}(u, v) = D(u, v) + deg(u) + deg(v)$, The Dd -radius, denoted by $r^{Dd}(G)$ is the minimum Dd -eccentricity among all vertices of G . That is $r^{Dd}(G) = \min\{e^{Dd}(G) : v \in V(G)\}$. Similarly the Dd -diameter, $D^{Dd}(G)$ is the maximum Dd -eccentricity among all vertices of G . We observe that for any two vertices u, v of G , We have $d(u, v) \leq D^{Dd}(u, v)$. The equality holds if and only if u, v are identical. If G is any connected graph then the Dd -distance is a metric on the set of vertices of G . We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$. The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

P. Poomalai et al was introduced the concept of radio analytic mean labeling in 2019. We are introduced the concept of radio analytic mean Dd -distance. The radio analytic labeling is a function $f: V(G) \rightarrow \mathbb{N}$ such that $D^{Dd}(u, v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(G)$. We are introducing the radio analytic mean Dd -distance number of some Modern graphs.

Theorem 1.1

The Radio analytic mean Dd -distance number of a Closed Helm graph CH_n ,

$$ramn^{Dd}(CH_n) = 2n + 2 \text{ for all } n.$$

Proof

Let $V(CH_n) = \{v_0, v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E(CH_n) = \{v_0v_i, v_iu_i, v_iv_{i+1}, u_iu_{i+1} | 1 \leq i \leq n\}$ be the edge set.

$$\text{The } Dd \text{ -distance } D^{Dd}(v_0, v_i) = 2n + 6, D^{Dd}(v_0, u_i) = 2n + 6,$$

$$D^{Dd}(v_i, v_j) = 2n + 8, D^{Dd}(u_i, u_j) = 2n + 6,$$

$$D^{Dd}(u_i, v_j) = 2n + 7, 1 \leq i \leq n, 2 \leq j \leq n - 1, i \neq j.$$

Obviously, $diam^{Dd}(CH_n) = 2n + 8$.

By the radio analytic mean Dd -distance condition is

$$D^{Dd}(u, v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(G),$$

For every pair of vertices (u, v) where $u \neq v$.

$$\text{Fix } f(v_0) = 2$$

Now,

$$D^{Dd}(v_0, v_1) + \left\lceil \frac{|f(v_0)^2 - f(v_1)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(CH_n)$$

$$\Rightarrow \left\lceil \frac{|(2)^2 - f(v_1)^2|}{2} \right\rceil \geq 3$$

Therefore, $f(v_1) = 3$

$$D^{Dd}(v_1, v_5) + \left\lceil \frac{|f(v_1)^2 - f(v_5)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(CH_n),$$

$$\Rightarrow \left\lceil \frac{|(3)^2 - f(v_5)^2|}{2} \right\rceil \geq 1$$

Therefore, $f(v_5) = 7$

Therefore, $f(v_1) = 3, f(v_2) = 4, f(v_3) = 5$, also, $f(v_4) = 6$,

So, $f(v_i) = i + 2, 1 \leq i \leq n$

Therefore, $f(v_n) = n + 2$.

$$D^{Dd}(v_0, u_1) + \left| \frac{|f(v_0)^2 - f(u_1)^2|}{2} \right| \geq 1 + \text{diam}^{Dd}(CH_n)$$

$$\Rightarrow \left| \frac{|(2)^2 - f(u_1)^2|}{2} \right| \geq 3$$

Therefore, $f(u_1) = n + 3$

$$D^{Dd}(u_1, u_2) + \left| \frac{|f(u_1)^2 - f(u_2)^2|}{2} \right| \geq 1 + \text{diam}^{Dd}(CH_n)$$

$$\Rightarrow \left| \frac{|(n+3)^2 - (f(u_2))^2|}{2} \right| \geq 3$$

Therefore, $f(u_2) = n + 4$

Therefore, $f(u_1) = n + 3, f(u_2) = n + 4$, also, $f(u_3) = n + 5$

So, $f(u_i) = n + i + 2, 1 \leq i \leq n$, Therefore, $f(u_n) = 2n + 2$.

Hence, $\text{ramn}^{Dd}(CH_n) \leq 2n + 2 \dots \dots \dots (1)$

Since CH_n has $2n+1$ vertices it requires $2n+1$ distinct labels. Also by the radio analytic mean Dd -distance condition 1 labels between 1 and n are forbidden.

$$\text{ramn}^{Dd}(CH_n) \geq (2n + 1) + 1$$

$$\geq 2n + 2 \dots \dots \dots (2)$$

From (1) and (2)

Hence, $\text{ramn}^{Dd}(CH_n) = 2n + 2$ for all n .

Theorem 1.2

The Radio analytic mean Dd -distance number of a Double Wheel graph,

$\text{ramn}^{Dd}(W_{n,n}) = 4n - 4$ for all n .

Proof

Let $V(W_{n,n}) = \{v_0, v_1, v_2, v_3, \dots, v_n\} \cup \{u_1, u_2, u_3, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E(W_{n,n}) = \{v_0v_i, v_iv_{i+1}, v_0u_i, u_iu_{i+1} | 1 \leq i \leq n\}$ be the edge set.

The Dd -distance $D^{Dd}(v_0, v_i) = 3n + 3, D^{Dd}(v_i, u_j) = 2n + 6,$

$$D^{Dd}(v_0, u_i) = 3n + 3, D^{Dd}(v_i, v_j) = n + 6, D^{Dd}(u_i, u_j) = 2n + 6, 1 \leq i \leq n, 1 \leq j \leq n.$$

Obviously, $diam^{Dd}(W_{n,n}) = 3n + 3.$

By the radio analytic mean Dd -distance condition is

$$D^{Dd}(u, v) + \left\lceil \frac{|f(u)^2 - f(v)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(G)$$

For every pair of vertices (u, v) where $u \neq v.$

Fix $f(v_0) = 1$

Now

$$D^{Dd}(v_0, v_1) + \left\lceil \frac{|f(v_0)^2 - f(v_1)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(W_{n,n})$$

$$\Rightarrow \left\lceil \frac{|(1)^2 - (f(v_1))^2|}{2} \right\rceil \geq 1$$

$$\therefore f(v_1) = 2n - 3$$

$$D^{Dd}(v_1, v_2) + \left\lceil \frac{|f(v_1)^2 - f(v_2)^2|}{2} \right\rceil \geq 1 + diam^{Dd}(W_{n,n})$$

$$\Rightarrow \left\lceil \frac{|(2n-3)^2 - f(v_2)^2|}{2} \right\rceil \geq 2n - 2$$

$$\therefore f(v_2) = 2n - 2$$

Therefore, $f(v_1) = 2n - 3, f(v_2) = 2n - 2,$ So $f(v_i) = 2n + i - 4, 1 \leq i \leq n$

Therefore, $f(v_n) = 3n - 4$

Hence, $ramn^{Dd}(W_{n,n}) \leq 3n - 4 \dots\dots\dots(1)$

Since $W_{n,n}$ has $2n+1$ vertices it requires $2n+1$ distinct labels. Also by the radio analytic mean Dd -distance condition $(2n-5)$ labels between 1 and n are forbidden.

$$ramn^{Dd}(W_{n,n}) \geq (2n + 1) + (2n - 5)$$

$$\geq 4n - 4 \dots \dots \dots (2)$$

From (1) and (2)

Hence, $ramn^{Dd}(W_{n,n}) = 4n - 4$ for all n .

Theorem 1.3

The Radio analytic mean Dd -distance number of a Shell graph $C(n, n - 3)$,

$$ramn^{Dd}(C(n, n - 3)) = 2n - 6, n \geq 7.$$

Proof

Let $V(C(n, n - 3)) = \{w_0, v_1, v_2, v_3, \dots, v_{n-1}\}$ be the vertex set and $E(C(n, n - 3)) = \{w_0v_i, v_iv_{i+1}, 1 \leq i \leq n - 1\}$ be the edge set.

The Dd -distance $D^{Dd}(w_0, v_i) = 2n, 1 \leq i \leq n, D^{Dd}(v_i, v_j) = n + 4, D^{Dd}(v_i, v_k) = n + 5, v_i, v_k$ are intermediate vertices.

Obviously, $diam^{Dd}(C(n, n - 3)) = 2n$.

By the radio analytic mean Dd -distance condition is

$$D^{Dd}(u, v) + \left| \frac{|f(u)^2 - f(v)^2|}{2} \right| \geq 1 + diam^{Dd}(G),$$

For every pair of vertices (u, v) where $u \neq v$.

Fix $f(w_0) = 1$

$$D^{Dd}(w_0, v_1) + \left| \frac{|f(w_0)^2 - f(v_1)^2|}{2} \right| \geq 1 + diam^{Dd}(C(n, n - 3))$$

$$\Rightarrow \left| \frac{|(1)^2 - f(v_1)^2|}{2} \right| \geq 1$$

$$\therefore f(v_1) = n - 4$$

$$D^{Dd}(v_1, v_2) + \left| \frac{|f(v_1)^2 - f(v_2)^2|}{2} \right| \geq 1 + diam^{Dd}(C(n, n - 3)),$$

$$\Rightarrow \left| \frac{|(n-4)^2 - f(v_2)^2|}{2} \right| \geq n - 2$$

$$\therefore f(v_2) = n - 3$$

Therefore, $f(v_1) = n - 4, f(v_2) = n - 3, \dots, f(v_i) = n - 1 + i - 4, 1 \leq i \leq n - 1$

Therefore, $f(v_{n-1}) = 2n - 6$.

Hence, $ramn^{Dd}((C(n, n - 3))) \leq 2n - 6 \dots \dots \dots (1)$

Since G_n has n vertices it requires n distinct labels. Also by the radio analytic mean Dd -distance condition $(n-6)$ labels between 1 and n are forbidden.

$$\begin{aligned} ramn^{Dd}(C(n, n-3)) &\geq (n) + (n-6) \\ &\geq 2n-6 \dots \dots \dots (2) \end{aligned}$$

From (1) and (2)

Hence, $ramn^{Dd}(C(n, n-3)) = 2n-6, n \geq 7$.

Note: $ramn^{Dd}(C(n, n-3)) = n$, if $n = 3, 4, 5, 6$.

Theorem 1.4

The Radio analytic mean Dd -distance number of a Sun Flower graph Sf_n ,

$$ramn^{Dd}(Sf_n) = 3n-5 \text{ for all } n.$$

Proof

Let $V(Sf_n) = \{v_0, v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E(Sf_n) = \{v_0v_i, v_iu_i, v_iv_{i+1}, u_iv_{i+1} \mid 1 \leq i \leq n\}$ be the edge set.

The Dd -distance $D^{Dd}(v_0, v_i) = 3n+4, D^{Dd}(v_0, u_i) = 3n+2$,

$$D^{Dd}(v_i, v_j) = 2n+8, D^{Dd}(u_i, u_j) = 2n+4,$$

$$D^{Dd}(u_i, v_j) = 2n+6, 1 \leq i \leq n, 2 \leq j \leq n-1, i \neq j.$$

Obviously, $diam^{Dd}(Sf_n) = 3n+4$.

By the radio analytic mean Dd -distance condition is

$$D^{Dd}(u, v) + \left| \frac{|f(u)^2 - f(v)^2|}{2} \right| \geq 1 + diam^{Dd}(G),$$

For every pair of vertices (u, v) where $u \neq v$.

$$Fix f(v_0) = 1$$

Now,

$$D^{Dd}(v_0, v_1) + \left| \frac{|f(v_0)^2 - f(v_1)^2|}{2} \right| \geq 1 + diam^{Dd}(Sf_n)$$

$$\Rightarrow \left| \frac{|(1)^2 - f(v_1)^2|}{2} \right| \geq 1$$

Therefore, $f(v_1) = n-4$

$$D^{Dd}(v_1, v_2) + \left| \frac{|f(v_1)^2 - f(v_2)^2|}{2} \right| \geq 1 + diam^{Dd}(Sf_n),$$

$$\Rightarrow \left\lceil \frac{|(n-4)^2 - f(v_2)^2|}{2} \right\rceil \geq n - 3$$

$$\text{Therefore, } f(v_2) = n - 3$$

Therefore, $f(v_1) = n - 4, f(v_2) = n - 3, f(v_3) = n - 2$, also, $f(v_4) = n - 1$,

So, $f(v_i) = n + i - 5, 1 \leq i \leq n$

Therefore, $f(v_n) = 2n - 5$.

$$D^{Dd}(v_0, u_1) + \left\lceil \frac{|f(v_0)^2 - f(u_1)^2|}{2} \right\rceil \geq 1 + \text{diam}^{Dd}(Sf_n)$$

$$\Rightarrow \left\lceil \frac{|(1)^2 - f(u_1)^2|}{2} \right\rceil \geq 3$$

$$\text{Therefore, } f(u_1) = 2n - 4$$

$$D^{Dd}(u_1, u_2) + \left\lceil \frac{|f(u_1)^2 - f(u_2)^2|}{2} \right\rceil \geq 1 + \text{diam}^{Dd}(Sf_n)$$

$$\Rightarrow \left\lceil \frac{|(2n-4)^2 - f(u_2)^2|}{2} \right\rceil \geq n + 1$$

$$\text{Therefore, } f(u_2) = 2n - 3$$

Therefore, $f(u_1) = 2n - 4, f(u_2) = 2n - 3$, also, $f(u_3) = 2n - 2$

So, $f(u_i) = 2n + i - 5, 1 \leq i \leq n$, Therefore, $f(u_n) = 3n - 5$.

Hence, $\text{ramn}^{Dd}(Sf_n) \leq 3n - 5 \dots\dots\dots(1)$

Since Sf_n has $2n+1$ vertices it requires $2n+1$ distinct labels. Also by the radio analytic mean Dd -distance condition $n-6$ labels between 1 and n are forbidden.

$$\text{ramn}^{Dd}(Sf_n) \geq (2n + 1) + n - 6$$

$$\geq 3n - 5 \dots\dots\dots(2)$$

From (1) and (2)

Hence, $\text{ramn}^{Dd}(Sf_n) = 3n - 5, n \geq 7$.

Note: $\text{ramn}^{Dd}(Sf_n) = 2n + 1, \text{ if } n = 3, 4, 5, 6$.

2. CONCLUSION

In this paper we studied the Radio analytic mean Dd -distance graphs, which involves Dd -distance and diameter. We computed the Radio analytic mean Dd -distance number by using in some Modern graphs and radio analytic mean number depends on the distance constraints.

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