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Research Paper

RESTRAINED SQUARE FREE DETOUR NUMBER OF A GRAPH

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ABSTRACT
In this paper, we define and determine the results on restrained square free detour number of some standard graphs and special graphs. Furthermore, we discuss relationship between the detour number and restrained square free detour number.The bounds of the restrained square free detour set of a graph are also established.

KEY WORDS−detour number, square free detour number, restrained detour number.

I. INTRODUCTION

In this paper, a graph $G$is considered to be a finite, undirected and connected graph of order $n(n \geq 2)$ with neither loops nor multiple edges. Let $D(x,y)$ be the longest path in G and an $x-y$ path of $D(x,y)$is called $x-y$ detour. The parameters on detour concept were developed by Chartrand [2].The detour concept was extended to triangle free detour concept by S. Athisayanathan et al. [1].The detour concept was applied in domination by number of authors. The detour domination number was studied and extended to restrained detour domination by number of authors in [4].The restrained concept was extended from geodetic to monophonic number. For any two vertices u, v in a connected graph $G(x,y)$, the $x-y$ path P is $x-y$ triangle free path if no three vertices of P induce a triangle. The triangle free detour distance $D_{\Delta f}(x-y)$ is the length of a longest $x-y$ triangle free path in G. In this paper, we introduce the restrained square free detour number denoted by$dn^{r}_{\square f}(G)$.The restrained square free detour number of some standard graphs and special graphs are determined.

II. PRILLIMINARIES

The following theorems and definitions are used in the sequel.

Theorem 0.1.[3] For any connected graph $G, 2 \leq dn(G) \leq n$.

Theorem 0.2.[3] Everyend-vertex of a non-trivial connected graph G belongs to every detour set of G.

Theorem 0.3.[3] If T is a tree with k end-vertices, then

$$n(T) = k.$$

Definition 0.4.The Helm H_n is obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle C_n.

Definition 0.5. The closed HelmCH_n is a graph obtained from a Helm H_n by joining each pendent vertex to form a cycle.

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Definition 0.6. The Fan Graph is the graph $P_n + K_1$, where $P_n: u_1 u_2 \dots u_n$ is a Path and $V(K_1) = u$.

Definition 0.7. Aprism graph$Y_n$is the graph obtained from a Helm by joining each pendent vertex to the central vertex of the Helm.

Definition 0.8. The Dutch Windmill graph$D_n^{(m)}$ is the graph obtained by taking m copies of the cycle graph C_n with a vertex in common.

III.MAIN RESULTS

1. Restrained Square Free Detour Number of Standard Graphs

Definition 1.1. Let $G = (V, E)$ be a simple connected graph of order $n \geq 2$. A set of vertices S^* is called restrained square free detour set in G if S^* is a square free detour set such that either $V = S^*$ or the induced subgraph$< S^* >$has no isolated vertices. The minimum cardinality ofa restrained square free detour number of G is called the restrained square free detour number of G and is denoted by $dn_{\square f}^r(G)$.

Example 1.2. For the graph G shown in Fig. 1, the set $S_1^* = \{u_1, u_5, u_6, u_3, u_8, u_9\}$ is a minimum restrained square free detour set and $S = \{u_1, u_5, u_3, u_9\}$ is a minimum square free detour set for G and so $dn_{\square f}^r(G) = 6$ and $dn_{\square f}(G) = 4$. Here,$dn_{\square f}(G) < dn_{\square f}^r(G)$. Moreover, the sets of six vertices $S_2^* = \{u_2, u_5, u_6, u_3, u_8, u_9\}$, $S_3^* = \{u_1, u_5, u_6, u_4, u_8, u_9\}$ and $S_4^* = \{u_2, u_5, u_6, u_4, u_8, u_9\}$ are also the minimum restrained square free detour sets of G. Hence there can be more than one minimum restrained square free detour set for a graph G.

Theorem 1.3.*For any connected graph G, everyend-vertex andeverysupport vertex of G belong to every restrained square free detour set of G.*

Proof. Since every restrained square free detour set is also a detour set of G, the proof follows from the Theorem 0.2.

Theorem 1.4.*If G is a Path P_n, then*

$$dn_{\square f}^r(G) = \begin{cases} n & if\ 2 \leq n \leq 4 \\ 4 & if\ n \geq 5 \end{cases}$$

Proof. Let $G = P_n$ be a path of order n and S be a set of end-vertices of G.Since S is independent, it cannot be a minimum restrained square free detour set. Therefore, consider two cases.

Case 1. Let$n = 2, 3, 4$.Then $S^* = V(G)$. Hence $< S^* >$has no isolated vertices and so $dn_f^r(G) = n$.

Case 2. Let$n \geq 5$. Then $S^* = S \cup S^\#$, where $S^\#$ is the set of support vertices of $G$with $|S^*| = 4$. Here we find that $< S^* >$has no isolated vertices.Thus the end-vertices and their support vertices produce the minimum restrained detour set and so $dn_{\square f}^r(G) = 4$.

Theorem 1.5.*If G is a complete graph K_n, then $dn_f^r(G) = 2$.*

Proof. Let $G = K_n$ be a complete graph of order n. Let S^* be any set of two vertices of G. Then every vertex of G lies in the square free detour of the adjacent vertices. Since every pair of vertices is connected by an edge in K_n,$< S^* >$has no isolated vertices. Hence $dn_f^r(G) = 2$.

Theorem 1.6.*If G is a complete bipartite graph $K_{m,n}(2 \leq m \leq n)$, then $dn_{\square f}^r(G) = m + 1$.*

Proof. Let $G = K_{m,n}$be a complete bipartite graph with two partitions X and Y, where $|X| = m$ and $|Y| = n$. Let$S$ be a set of $m$vertices of X. Now, it is easy to verify that$|S| = m, S$ is independent. Therefore, consider $S^* = S \cup \{y\}$, where y is any vertex of $Y$with $|S^*| = m + 1$. Here $< S^* >$has no isolated vertices. Hence $dn_{\square f}^r(G) = m + 1$.

If G is a star S_n, then $dn_{\square f}^r(G) = dn(G) + 1$.

Proof. Let $G = S_n$be a star with $n - 1$ end-vertices. Then by Theorem 2.3, $S = \{x_i: 1 \leq i \leq n - 1\}$ where $x_i, x_2, \ldots, x_{n-1}$ are the end-vertices of S_n such that S is independent. Hence $dn(G) = n - 1$. By Theorem 1.3,S is a minimum detour set but nota minimum restrained square free detour set. Then by Theorem 1.3, S^* is the set of all vertices of $G$with $|S^*| = n$. Thus $dn_{\square f}^r(G) = n$. Hence $dn_{\square f}^r(G) = dn(G) + 1$.

IV. CONCLUSION

In this paper, we investigated the restrained square free detour number of some standard graphs and special graphs like helm, closed helm, fan, prism, Dutch windmill graph. Further investigation is open for any other class of graphs.

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