ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

The Feasibility of Matrix Riccati-Type Dynamical Systems on Time Scales

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Abstract

This paper introduces the notions of realizability and minimum realizability criteria for a matrix dynamical system of first order, considering a zero initial state, on time scales.

1. Introduction

The significance of matrix dynamical systems resembling the Riccati equation is widely acknowledged, as they find applications in diverse fields of applied mathematics, including control systems, dynamic programming, optimal filters, quantum mechanics, and systems engineering. This paper is centred around two principal goals: [1] advancing the theory and methodologies for solving dynamical systems on time scales, and [2] investigating realizability techniques. The primary emphasis of this paper revolves around matrix dynamical systems of Riccati-like nature on time scales, specifically in the format of[3]:

$$X^{\Delta}(t) = A(t)X(t) + X(t)B(t) + \mu(t)A(t)X(t)B(t) + C(t)U(t)D^{*}(t), \quad X(t_0) = X_0 \qquad (1.1)$$

$$Y(t) = K(t)X(t)L^{*}(t) \qquad (1.2)$$

If the time scale T = R, the system (1.1) becomes Sylvester matrix differential system of the form

$$X^{\Delta}(t) = A(t)X(t) + X(t)B(t) + C(t)U(t)D^{*}(t) \qquad (1.3)$$

$$\Delta X(t) = A(t)X(t) + X(t)B(t) + A(t)X(t)B(t) + C(t)U(t)D^{*}(t) \qquad (1.4)$$

$$X(t+1) = A_1(t)X(t)B_1(t) + C(t)U(t)D^{*}(t) \qquad (1.5)$$

2. Preliminaries

Significant progress has [4]taken place since 1988 in consolidating the theories of differential equations[5] and difference equations by achieving parallel [6]outcomes within the framework of time scales[7]. For more comprehensive insights, you can consult the references provided in books [8].

Definition 2.1

A nonempty closed subset of R is called a time scale[9]. It is denoted by T. By an interval we mean the intersection[10] of the given interval with a time scale[11].

ISSN PRINT 2319 1775 Online 2320 7876

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Theorem 2.1 Assume $f : T \to R$ is a function and let $t \in T^k$. Then we have the following:

(i) If f is differentiable at t, then f is continuous at t.

(ii) If f is continuous at t and t is right-scattered, then f is differentiable at t with

$$f^{\Delta}(t) = \frac{f(\sigma(t)) - f(t)}{\mu(t)}.$$

(iii) If t is right-dense, then f is differentiable at t iff the limit

$$\lim_{t \to s} \frac{f(t) - f(s)}{t - s}$$

Exists as a finite number. In this case

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f(t) - f(s)}{t - s}$$

(iv) If f is differentiable at t, then

$$f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t).$$

Result 2.1 If $A, B \in R$ are matrix-valued functions on T, then

(i) $\phi_0(t,s) \equiv I$ and $\phi_A(t,t) \equiv I$,

(ii) $\phi_A(\sigma(t), s) \equiv (I + \mu(t)A(t))\, \phi_A(t,s)$;

(iii) $\phi_A^{-1}(t,s) \equiv \phi^{*}_{\ominus A^{*}}(t,s)$;

(iv) $\phi_A(t,s) = \phi_A^{-1}(s,t) = \phi^{*}_{\ominus A^{*}}(s,t)$; (v) $\phi_A(t,s)\phi_A(s,r) = \phi_A(t,r)$;

(vi) $\phi_A(t,s)\phi_B(t,s) = \phi_{A \oplus B}(t,s)$ if $\phi_A(t,s)$ and B(t) commute.

Theorem 2.2 [2] Let $A \in R$ be an $n \times n$-matrix-valued function on T and suppose that $f : T \to R^n$ is rd-continuous. Let $t_0 \in T$ and $y_0 \in R^n$. Then the initial value problem

$$y^{\Delta}(t) = A(t)\,y(t) + f(t), \ \ y(t_0) = y_0,$$

Has a unique solution $y : T \to R$ .Moreover, this solution is given by

$$y(t) = \phi_A(t, t_0)\, y_0 + \int_{t_0}^{t} \phi_A(t, \sigma(\tau))\, f(\tau)\Delta\tau$$

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The Kronecker product has the following properties and rules.

1. $(A \otimes B)^{*} = A^{*} \otimes B^{*}$,
2. $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$ (provided A and B are invertible)
3. The mixed product rule $(A \otimes B)(C \otimes D) = (AC \otimes BD)$, Provided the dimensions of the matrices are such that the various expressions exist.
4. $\| A \otimes B \| = \| A \| \, \| B \|$
5. $\mathrm{Vec}(AYB) = (B^{*} \otimes A)\, VecY$
6. If A and B are matrices both of order $n \times n$ then
   (i) $Vec(AX) = (I_n \otimes A)$ VecX
   (ii) $Vec(XA) = (A^{*} \otimes I_n)$ VecX

Let A and B are rd-continuous matrices on time scale T, then

$$(A \otimes B)^{\Delta}(t) = A^{\Delta}(t) \otimes B(t) + A(\sigma(t)) \otimes B^{\Delta}(t)$$

Now by applying the Vec operator to the Δ-differentiable matrix dynamical system (1.1) also the output equation (1.2) and using Kronecker product properties, we have

$$Z^{\Delta}(t) = G(t)Z(t) + [D \otimes C]\hat{U}(t); \qquad Z(t_0) = Z_0; \tag{2.1}$$

$$\hat{Y}(t) = [L \otimes K]Z(t) \tag{2.2}$$

Where, Z(t) = Vec X(t), $\hat{U}(t) =$ Vec U(t) , $\hat{Y}(t) =$ VecY (t) and

G(t) $= [B^{*} \otimes I + I \otimes A + \mu(t)(B^{*} \otimes A)]$, is a $n^2 \times n^2$ matrix. Let A(t) and B(t) be regressive and rd-continuous.

From the definition of Kronecker product G: $T^k \to R^{n2}$ is regressive and rd-continuous.

System (2.1) and (2.2) is called the Kronecker product system associated with (1.1) and (1.2).

Remark 2.1 It is easily seen that, if X(t) is the solution of (1.1) then VecX(t)= Z(t) is the solution of (2.1) and vice-versa.

Now we confine our attention to corresponding homogeneous matrix dynamical system on time scales (2.1) given by

$$Z^{\Delta}(t) = G(t)Z(t) \tag{2.3}$$

3. MAIN RESULTS

ISSN PRINT 2319 1775 Online 2320 7876

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In this section, [12]we discuss realizability and minimal realizability for the matrix dynamical systems on time scales[13]

Definition 3.1 The Δ-differential systems S_1 given by (2.1) is said to be completely controllable if for t_0, any initial state $Z(t_0) = Z_0$ and any given final state Z_f there exists a finite time $t_1 > t_0$ and a control $\hat{U}(t), t_0 \le t \le t_1$ such that $Z(t_1) = Z_f$.

Lemma 3.1 [12]The time scale dynamical system S_1 is completely controllable on the closed interval $J = [t_0, t_1]$ if and only if the $n^2 \times n^2$ symmetric controllability matrix

$$V(t_0, t_1) = \int_{t_0}^{t_1} \phi(t_0, \sigma(s))(D \otimes C)(s)(D \otimes C)^*(s)\phi^*(t_0, \sigma(s))\Delta s \qquad (3.1)$$

Where $\phi(t, s)$ is defined in (2.4), is non-singular. In this case the control

$$\hat{U}(t) = -(D \otimes C)^*(t)\phi^*(t_0, \sigma(s))V^{-1}(t_0, t_1)\{Z_0 - \phi(t_0, t_1)Z_f\} \qquad (3.2)$$

Defined on $t_0 \le t \le t_1$, transfers $Z(t_0) = Z_0$ to $Z(t_1) = Z_f$.

Theorem3.1 A realization exists for a matrix R(t,s) if and only if it can be expressed in the form

$$R(t, s) = P(t)Q(s) \qquad (3.7)$$

Where P and Q are matrices having finite dimensions

Proof: Suppose R(t, s) posses a realization, then (3.6) exists and

$$\begin{aligned} R(t,s) &= (L \otimes K)\phi(t, \sigma(s))(D \otimes C)(s) \\ &= (L \otimes K)\{\phi_2(t, \sigma(s)) \otimes \phi_1(t, \sigma(s))\}(D \otimes C)(s) \\ &= (L \otimes K)\{X_2(t)X_2^{-1}(\sigma(s)) \otimes X_1(t)X_1^{-1}(\sigma(s))\}(D \otimes C)(s) \\ &= (L \otimes K)\{(X_2(t) \otimes X_1(t))(X_2^{-1}(\sigma(s)) \otimes X_1^{-1}(\sigma(s))\}(D \otimes C)(s) \\ &= P(t)Q(s). \end{aligned}$$

Where,

X_1 and X_2 are fundamental matrices of the systems $X^\Delta(t) = A(t)X(t)$ and $X^\Delta(t) = B^*(t)X(t)$ respectively,

$P(t) = (L \otimes K)\{(X_2(t) \otimes X_1(t))$ and $Q(s) = (X_2^{-1}(\sigma(s)) \otimes X_1^{-1}(\sigma(s))\}(D \otimes C)(s)$. Hence (3.7) is certainly a necessary condition.

Conversely, if (3.7) holds $R(t, s) = P(t)Q(s) = P(t)I_{n2}Q(s)$ this implies that $\phi(t, s) = I_{n^2}$, then a realization of R(t, s) is $\{O_{n2}, Q(t), P(t)\}$, where O_{n2} denotes an $n^2 X n^2$ null matrix.

4. References

1. Alexander. G. Kronecker products and matrix calculus; with applications, Ellis Hord wood Ltd., England, (1981).

2. Appa Rao B. V, Prasad K A S N V, Controllability and observability of Sylvester Matrix Dynamical Systems on time scales, Kyungpook, Math. J. 56(2016),529-539.

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 10, Spl Iss 1, 2021

3. Appa Rao B. V, Minimum energy control of delta differentiable positive matrix Sylvester dynamical systems, Int.J.Chem.Sci.,14,2016, 751-761.

4. Bhoner.M and Peterson.A, Dynamic equations on time scales, Birkhauser, Boston, (2001).

5. Bhoner.M and Peterson.A, Advances in Dynamic equations on time scales, Birkhauser, Boston, (2003).

6. Davis John M, Gravagre Ian A, Jackson Billy. J and Marks Robert. J, Controllability, observability, realizability and stability of dynamic linear systems, Electronic journal of differential equations Vol 2009 No.37, (2009), 1-32.

7.Fausett L.V, on Sylvester matrix differential equations, Analytical and numerical solutions, International Journal of Pure and Applied Mathematics, Vol.53, No.1, (2009), 55-68.

8.Hilger. S, Analysis on measure chains a unified approach to continuous and discrete calculus, Results Math. 18, (1990), 18–56.

9.Ho Y. C, Kalman R. E, and Narendra K. S. Controllability of linear dynamical systems. Contributions to Differential Equations (1963), 189–213.

10.Lakshmikantam.V, Deo S.G, Method of variation of parameters for Dynamical systems, Gordan and Breach Scientific Publishers, (1998).

11. Murty M.S.N, Appa Rao B.V, Controllability and observability of Matrix Lyapunov

systems, Ranchi Univ.Math.Journal Vol.No.32, (2005), 55-65.

12. Murty M.S.N , Appa Rao B.V,and Suresh Kumar G, Controllability, Observability and realizability of Matrix Lyapunov systems, Bull.Korean Math.Soc.,43, No.1, (2006), 149-159.

13. Murty K.N, Anand P.V.S, Lakshmi Prasannam V, First order difference system-existence and uniqueness, Proceedings of the American Mathematical Society, 125(12), (1997), 3533-3539.