

# Integrated Inventory Model With Controllable Lead Time Involving Investment For Quality Improvement In Supply Chain System With Environmental Sustainability

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## ABSTRACT

In this study, we consider integrated inventory model with controllable lead time involving investment for quality improvement in supply chain system. The lead time is identical for all buyers as well as it can be shortened by paying an additional crashing cost which is exponentially function of lead time. To find out an optimal inventory strategy that can minimize the value of the integrated total relevant cost with the concern of environmental sustainability. Finally, a numerical example is presented to illustrate the proposed model.

## INTRODUCTION

Some time ago, the companies can be obtained the competitive advantage by strengthening their competitiveness. As a result, we can use the integrated inventory model to obtain minimum the total relevant cost for both vendor and buyer. The just-in-time approach to productivity demands that small lots to be run in production. This can be only achieved if the setup time is reduced. The ability to reduce setup cost over time can be explained in the terms of the learning curve.

Inventory Control is important in Supply Chain Management. Inventories play an extremely important role in a nation's economy. In recent Years, most inventory problems have their focus on the integration between long-term strategic partnerships between the vendor and the buyer is advantageous for the two parties regarding costs, and therefore profits since both parties, to achieve improved benefits, cooperate and share information with each other. Therefore, several researchers have shown that the vendor and the buyer can achieve their own minimal total cost, or increase their mutual benefit through strategic cooperation with each other.

In the production environment, lead time plays an important role in today's logistics management. Define as the time that elapses between the placements of another into inventory. Lead time may influence customer service and impact inventory costs. As the Japanese example of just-in time-production has shown, consequently reducing lead time may increase productivity and improve the competitive position of the company. Although lead time can be constant or variable, it is often treated as a prescribed parameter in most of the studies. Therefore, the lead time crashing cost function is a piecewise linear function.

The number of advantages have been associated the efforts of control of the lead time (which is a goal of JIT inventory management philosophies that emphasizes high quality and keeps low inventory level and lead time to a practical minimum). Lead time management is a significant issue in production and operation management. In many practical situations, lead time can be reduced using an added crashing cost. In other words, lead time is controllable.

The crashing of lead time mainly consists of the following components: order preparation, order transit, supplier lead time and delivery time.

Supply Chain Management has taken a very important and Critical role for any company increasing globalization and competition in the market. A supply chain model (SCM) is a network of suppliers, producers, distributors and customers which synchronizes a series of interrelated business process in order to have

- Optimal procurement of raw materials from nature.
- Transportation of raw materials into a warehouse.
- Production of goods in the production centre, and
- Distribution of these finished goods to retailers for sales to the customers.

In the current supply chain management (SCM) environment companies are using JIT production to gain and maintain a competitive advantage. JIT requires a spirit of cooperation between the buyer and the vendor, and it has been shown that forming a partnership between the buyer and the vendor is helpful in achieving tangible benefits for both parties. In current year, enter prices business models are different from the past due to globalization and information. The enter prices have to face many predicaments in internationalization, so there will be the emergence of the concept of supply chain. Compared to the traditional business model, the supply chain system can integrate the upstream and downstream companies between enter prices and use the resources more efficiently to create more profit.

The collaboration concept has become an accept practice in many successful global business organizations and provides economic advantages for both a vendor and a buyer. As a certified supplier, the vendor needs to perfect production process by its efforts to improve the buyer's operational efficiency and maintain a win-win relationship with the buyer.

In competitive markets products are pushed faster and faster to customers along supply chain resulting in faster generation of waste and depletion of natural resources. The word "Reverse logistics" as a business term. The General definition is "Reverse Logistics is such activity which helps to continue an environmental effective policy of firms with reuse of necessary materials, remanufacturing, and with reduction of amount of necessary materials". This reverse activity includes take back of damaged products, renewal and enlargement of inventories through product take back remanufacturing of packaging materials, reuse of containers and renovation of product, and handling of obsolete appliances.

In this complex environment, successful companies have denoted considerable attention to reducing inventory cost and improving quality could reduce waste, in other words, cut the cost. Therefore, in this present study inventory model with controllable lead time involving investment for quality improvement in supply chain system and intend to proposes a simple solution procedure to search the optimal production and process quality that can minimise the integrated total relevant cost.

Today's supply chain environment requires a new spirit of cooperation between the single buyer and the single vendor. We consider integrated inventory model with controllable lead time involving investment for quality improvement in supply chain system. The lead time is identical for all buyers as well as it can be shortened by paying an additional crashing cost which is exponentially function of lead time. And lead time expressed in weeks. To find

out an optimal inventory strategy that can minimize the value of the integrated total relevant cost for both the single buyer and the single vendor. Finally, a numerical example is presented to illustrate the proposed model.

## REVIEW OF THE RELATED LITERATURE

Goyal (1976) is among the first researchers who studies integrated inventory model for single vendor buyer system. He introduces a model for situation in which vendor produces a lot based on an infinite production rate and transfers it to the buyer by a lot-for-lot policy. In 1986, Banerjee assumed that the vendor manufactures at a finite rate and considered a joint economic-lot-size model in which a vendor produces to order for a buyer on a lot-for-lot basis. Goyal (1988) relaxed the lot-for-lot policy and suggested that vendors economic production quantity should be an integer multiple of buyer purchase quantity. Pan et al. (2002) improved Goyals (1988) model by considering lead time as a controllable factor in the model and obtained a lower joint total expected cost and shorter lead time. Liao et al. (1991) addressed a probabilistic inventory model in which the lead time is a decision variable. Ben-Daya et al. (1994) extended Liao et al.(1991) model by allowing both the lead time and the order quantity as decision variables. Ha et al.(1997) proposed an integrated lot-splitting model of facilitating multiple shipments in small lots. Hoque et al.(2000) proposed an integrated production-inventory system involving the capacity of transport equipment. Yang et al.(2000) presented an integrated model considering economic ordering policy of deteriorated item. Nieuwenhuysc et al. (2006) found that lot splitting policies have benifited both the vendor and the buyer. Huang et al. (2010) presented the permissible delay in payment problem in a single vendor and a single buyer integrated inventory model. Thasi (2011) developed a production and shipment model for a system that incorporates learning effect and deteriorating items and to derive an optimal joint total cost from the integrated perspective of both vendor and buyer. Uthayakumar et al. (2012) proposed a model that integrates the single vendor single buyer problem with order processing cost reduction and process mean.

Pan et al. (2002a) assumed the crash cost is a function of both the order quantity and the received lead time, and then established inventory models with fixed and variable lead time crash cost. Pan et al. (2005) investigated an integrated inventory system in which shortage is allowed and both lead time and backordering are negotiable. Chang et al. (2006) proposed integrated vendor – buyer cooperative inventory models with controllable lead time and ordering cost reduction. Vijayashree et al. (2014) developed an integrated inventory model with controllable lead time and setup cost reduction for both defective and non- defective items. Priyan et al. (2014) developed mathematical modelling for EOQ inventory system with advance payment and fuzzy parameters.

Ouyang et al. (2004) investigated the influence of ordering cost reduction on modified continuous review inventory systems involving variable lead time with partial backorders. Quyang et al. (2000) investigated the impact of quality improvement on the modified lot-size reorder point models involving variable lead time and partial backorders. Ouyang et al. (2002) extended Ouyang et al. (2000) model by investing in process quality improvement and setup cost reduction simultaneously. Uthayakumar et al. (2013) developed supply chain model with variable lead time under credit policy. Vijayashree et al. (2014) developed a two-stage supply chain model with selling price dependent demand and investment for

quality improvement. Ouyang et al. (2007) developed an integrated vendor-buyer inventory model with quality improvement and lead time reduction. Yang et al. (2004) developed an integrated inventory model involving deterministic variable lead time and quality improvement investment.

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## NOTATIONS

To establish the proposed model, the following notations are used:

- D** Buyer's expected demand rate in units per unit time.
- P** Vendor's productions rate in units per unit time,  $P > D$ .
- Q** Buyer's order quantity in units.
- A** Buyer ordering cost per order.
- S** Vendor's setup cost per setup.
- L<sub>s</sub>** Normal duration to arrive the items in buyer inventories.
- L<sub>e</sub>** Minimum duration to arrive the items in buyer inventories.
- C<sub>v</sub>** Unit production cost paid by the vendor.
- C<sub>b</sub>** Unit purchase cost by the buyer.
- L** Length of lead time.
- n** The number of shipments in which the product is delivered from the vendor to the buyer in one production cycle.
- r** Annual inventory holding cost per dollar invested in stocks.
- s** Vendor unit defective cost per defective item.
- θ<sub>1</sub>** Social cost from vechile emission.
- θ<sub>0</sub>** Original probability of the vendor's production process that can go out of control.
- I(θ)** Vendor's capital investments require for reducing the out of control probability from θ to θ<sub>0</sub>.
- i** Vendor's fractional opportunity cost of capital per unit time.
- TRC** Total relevant cost for the single vendor and the single buyer.
- μ** Proportion of demand returned ( $0 < \mu < 1$ ).
- δ** Proportion of waste produced per lot Q.

- a Fixed cost per trip(monetary unit).
- b Variable cost per unit transported per distance travelled.
- d Distance travelled.
- $\gamma$  Cost to dispose waste to the environment(mu/unit).

### ASSUMPTIONS

To develop the proposed model, we adopt the following assumptions:

1. The system consists of a single vendor and a single buyer for a single product in this model, and the inventory system deals with only one type item.
2. The buyer orders a lot of size  $Q$  and the vendor manufactures  $nQ$  with a finite production rate  $P$  ( $P > D$ ) at one set-up but ship quantity  $Q$  to the buyer over  $n$  times. The vendor incurs a set-up cost  $S$  for each production run and the buyer incurs an ordering cost  $A$  for each order of quantity  $Q$ .
3. There is vendor and buyer for a single product in this model.
4. The demand  $X$  during lead time  $L$  follows a normal distribution with mean  $\mu L$  and standard deviation  $\sigma\sqrt{L}$ .
5. The inventory is continuously reviewed. The buyer places the order when the on hand inventory reaches the reorder point  $R$ .
6. The buyer places the order when the inventory position reaches the reorder point  $R$ . the reorder point  $R =$  the expected demand during lead time+ safety stock. (i.e.)  $R = DL + K \sigma\sqrt{L}$ .
7. The extra cost incurred by the vendor will be transferred to the buyer if shortened lead time is requested.
8. If the buyer is not eager to add extra cost to control the lead time, he should obtain his items at exactly normal lead time and crashing cost is zero. Here, the buyer added crashing cost to control the lead time. Therefore, the lead time  $L$  should be within the interval  $L \in [L_e, L_s]$ . That is  $L_e < L < L_s$ .
9. The crashing cost were observed to grow with lead time by a proportion which can be approximated by an exponentially function of lead time. Therefore, the lead time crashing cost per order  $R(L)$ , is assumed to be an exponentially function of  $L$  and is defined as  $R(L) = 0$  if  $L = L_s$ .
10. The relationship between lot size and quantity is formulated as follows: while vendor is producing a lot, the process can go out-of-control with a given probability  $\theta$  each time another unit is produced. The process is assumed to be in control in the beginning of the production process. Once out-of-control, the process produces defective items and continues to do so until the entire lot is produced.
11. The out-of-control probability  $\theta$  is a decision variable.
12. Demand is known, constant and independent.
13. Single product case.
14. No shortage.

**MODEL DEVELOPMENT**

**INTEGRATED TOTAL COST (ITC)**

An integrated vendor-buyer inventory model in order to minimize the sum of the ordering cost, holding cost, setup cost, transportation cost, investment for quality improvement and crashing cost by simultaneously optimizing the optimal order quantity, process quantity, lead time and number of deliveries. Here the lead time crashing cost has been assumed to be an exponentially function of the lead-time length.

The joint total expected cost per unit time consists of the following elements,

$$\begin{aligned} \text{Ordering cost per unit time} &= \left( \text{cost per order} \right) \left( \begin{array}{c} \text{no. of orders} \\ \text{in} \\ \text{the inventory} \\ \text{period} \end{array} \right) \\ &= \frac{A}{T} \\ &= \frac{AD}{Q} \quad \text{where } T = \frac{Q}{D} \end{aligned}$$

Buyer’s holding cost per unit time

$$= \left( \frac{Q}{2} + K\sigma\sqrt{L} \right) rC_b$$

Lead time crashing cost per unit time

$$= \left( \frac{D}{Q} \right) R(L)$$

Vendor setup cost per year =  $\left( \frac{D}{nQ} \right) S$

Vendor’s holding cost per unit time:

Vendor’s average inventory is evaluated as the difference of the vendor’s accumulated inventory and the buyer’s accumulated inventory.

Vendor’s average inventory

$$\begin{aligned} &= \left\{ \left[ nQ \left( \frac{Q}{P} + (n-1) \frac{Q}{D} \right) - \frac{n^2 Q^2}{2P} \right] - \left[ \frac{Q^2}{D} (1 + 2 + \dots + n - 1) \right] \right\} \frac{D}{nQ} \\ &= \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] \end{aligned}$$

Vendor’s holding cost per unit time

$$= \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v$$

The integrated total cost per unit time for the single vendor and the single buyer integrated inventory system is given by

ITC(Q, L, n) = ordering cost + buyer’s holding cost + lead time crashing cost + vendor’s setup cost + vendor’s holding cost

$$= \frac{AD}{Q} + \left( \frac{Q}{2} + K\sigma\sqrt{L} \right) rC_b + \frac{D}{Q} R(L) + \left( \frac{D}{nQ} \right) S + \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v$$

$$= \frac{AD}{Q} + \frac{Q}{2} rC_b + K\sigma\sqrt{L} rC_b + \frac{D}{Q} R(L) + \left( \frac{D}{nQ} \right) S + \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v$$

$$ITC(Q, L, n) = \frac{D}{Q} \left( A + \frac{S}{n} + R(L) \right) + \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + K\sigma\sqrt{L} rC_b + \frac{Q}{2} rC_b$$

Defective item rework cost per unit time:

The expected number of defective items in a run of size nQ with a given probability of  $\theta$  that the process can go out of control is  $\frac{n^2}{2} Q^2 \theta$ . Thus, the defective cost per unit time is given by  $\frac{SnQD\theta}{2}$ .

Hence, the total cost incorporating the defective cost per year can be represented by

$$TC(Q, L, n) = ITC(Q, L, n) + \frac{SnQD\theta}{2}$$

Transportation cost per cycle =  $2a + bdQ + bd\mu Q$

Emission cost from transportation per cycle

$$= 2 \theta_1 \frac{d}{v} \text{ where the number '2' refers to a round trip.}$$

Waste produced by the inventory system per cycle is

$$= \gamma_0 + \gamma Q (\delta + \mu)$$

The total packaging cost per cycle is =  $(p + L)m$

The total cost per year

$$TC(Q, L, n) = ITC(Q, L, n) + \frac{SnQD\theta}{2} + 2a + bdQ + bd\mu Q + 2 \theta_1 \frac{d}{v} + (p + L)m + \gamma_0 + \gamma Q (\delta + \mu)$$

Based on the above equation, this study is an attempt to study the effect of investment on quality improvement.

The total relevant cost per year is

$$\begin{aligned}
 TRC(Q) &= TC(Q, L, n) + iq \ln \frac{\theta_0}{\theta} \\
 &= ITC(Q, L, n) + \frac{SnQD\theta}{2} + 2a + bdQ + bd\mu Q + 2\theta_1 \frac{d}{v} + (p + L)m + \gamma_0 + \gamma Q (\delta + \mu) + iq \ln \frac{\theta_0}{\theta} \\
 &= \frac{D}{Q} \left( A + \frac{S}{n} + R(L) \right) + \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + K\sigma\sqrt{L} rC_b + \frac{Q}{2} rC_b + \frac{SnQD\theta}{2} + 2a + bdQ + bd\mu Q + 2\theta_1 \frac{d}{v} + (p + L)m + \gamma_0 + \gamma Q (\delta + \mu) + iq \ln \frac{\theta_0}{\theta}
 \end{aligned}$$

The main objective is to determine the minimal optimal quantity.

$$\begin{aligned}
 \frac{\partial TRC(Q)}{\partial Q} &= \frac{\partial}{\partial Q} \left[ \frac{D}{Q} \left( A + \frac{S}{n} + R(L) \right) + \frac{Q}{2} \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + K\sigma\sqrt{L} rC_b + \frac{Q}{2} rC_b + \frac{SnQD\theta}{2} + 2a + bdQ + bd\mu Q + 2\theta_1 \frac{d}{v} + (p + L)m + \gamma_0 + \gamma Q (\delta + \mu) + iq \ln \frac{\theta_0}{\theta} \right] \\
 &= - \left( A + \frac{S}{n} + R(L) \right) + \frac{1}{2} \left[ \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + rC_b + SnD\theta \right] + bd + bd\mu + \gamma(\delta + \mu)
 \end{aligned}$$

∴ The necessary condition is  $\frac{\partial TRC(Q)}{\partial Q} = 0$

$$\begin{aligned}
 &\frac{1}{2} \left[ \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + rC_b + SnD\theta \right] + bd + bd\mu + \gamma(\delta + \mu) \\
 &= \frac{D}{Q^2} \left( A + \frac{S}{n} + R(L) \right)
 \end{aligned}$$

$$Q^2 = \frac{(A + \frac{S}{n} + R(L))D}{\frac{1}{2} \left[ \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + rC_b + SnD\theta \right] + bd + bd\mu + \gamma(\delta + \mu)}$$

$$Q = \sqrt{\frac{D(A + \frac{S}{n} + R(L))}{\frac{1}{2} \left[ \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + rC_b + SnD\theta \right] + bd + bd\mu + \gamma(\delta + \mu)}}$$

$$Q = \sqrt{\frac{2D(A + \frac{S}{n} + R(L))}{\left[ \left[ n \left( 1 - \frac{D}{P} \right) - 1 + \frac{2D}{P} \right] rC_v + rC_b + SnD\theta \right] + 2bd + 2bd\mu + 2\gamma(\delta + \mu)}}$$

**NUMERICAL EXAMPLE**

We consider the numerical example with the following data, D = 600 units/year, P = 2000 units/year, A = \$ 200/ order, r=0.2,



$C_v=70/\text{unit}$ ,  $C_b=100/\text{unit}$ ,  $S=\$ 1500 / \text{setup}$ ,  $n=3$ ,  $\theta=0.000021316$ ,  $\mu=1$ ,

$\delta=1$ ,  $\gamma=1$ ,  $R(L)=6$ ,  $b=0.5$

$$Q = \sqrt{\frac{2D\left(A + \frac{S}{n} + R(L)\right)}{\left[\left[n\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]rC_v + rC_b + SnD\theta\right] + 2bd + 2bd\mu + 2\gamma(\delta + \mu)}}$$

$$= \sqrt{\frac{2 \times 600 \left(200 + \frac{1500}{3} + 6\right)}{0.2 \left[\left(3 \left(1 - \frac{600}{2000}\right) - 1 + \frac{1200}{2000}\right) 70 + 100\right] + (1500 \times 3 \times 600 \times 0.000021316) + (2 \times 0.5 \times 600 \times 2) + (2 \times 1 \times 2)}}$$

$Q=25.47$  units/year

### CONCLUSION

An inventory model for two-stage supply chain is investigated. A supply chain single vendor and single buyer is considered. In this study, we consider integrated inventory model with controllable lead time involving investment for quality improvement in supply chain system. Lead time is an important element in any inventory management system. Industrial buyers often call it as lead time. Firm can shorten delivery times by storing inventory or having excess capacity. In many practical situations, lead time can be reduced at an added crashing cost: in other words, it is controllable. The numerical example is given to illustrate the benefit of coordination between single vendor and single buyer.

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