e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.12, 2022
Research Paper UGC CARE Listed (Group -I) Journal

THE UPPER COMPLEMENT CONNECTED EDGE GEODETIC NUMBER OF A GRAPH

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ABSTRACT—A complement connected edge geodetic set of G is called a minimal complement connected edge geodetic set of G if no proper subset of S is a complement connected edge geodetic set of G. The upper complement connected edge geodetic number $g_{cce}^{+}(G)$ is the maximum cardinality of a minimal complement connected edge geodetic set of G. Some general properties satisfied by this concept are studied connected graphs of order p ≥ 3 with $g_{cce}^{+}(G$ to be $p - 1$ is given. It is shown that for every pair of integers a and b with3 $\leq a \leq b$, there exists a connected graph G with $gcce(G) = a$ and $g_{cce}^{+}(G) = b$,where upper complement connected edge geodetic number of a graph.
Keywords—distance, edge geodetic number, complement connected edge geodetic number, upper complement connected edge geodetic number.

1. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite, undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology, we refer to [2]. For the neighborhood of the vertex v in G, $N(v) = \{u \in V(G) : uv \in E(G)\}$. The degree of a vertex v of a graph is $deg(v) = |N(V)|$. $\Delta(G)$ and $\delta(G)$ are the maximum and minimum degrees of the graph respectively. A vertex v is said to be universal vertex if $deg(v) = p - 1$. For $S \subseteq V(G)$, the induced subgraph $G[S]$ is the graph whose vertex set is S and whose edge set consists of all of the edges in E that have both endpoints in S. A vertex v is called an extreme vertex of a graph G if $G[N(v)]$ is complete. A vertex v in a connected graph G is said to be a semi-extreme vertex if $\Delta(G[N(v)]) = |N(v)| - 1$. Every semi-extreme vertex is extreme vertex of G that there are extreme vertices which are not a extreme vertex of G. A graph G is said to be semi-extreme graph if every vertex of G is a semi-complete vertex. A graph with at least two universal vertices is a semi-complete graph. Infact, there are semi-complete graph which hasno universal vertices. The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. A vertex x is said to lie on a $u - v$ geodesic P if x is a vertex of P includingthe vertices u and v. The eccentricity $e(v)$ of a vertex v in G is the maximum distancefrom $v$and a vertex of G. $e(v) = max\{d(v, u) : u \in V\}$ The minimum eccentricity among the vertices of G is the radius, $radG$ or $r(G)$ and the maximum eccentricity isits diameter, $diamG$. We denote $rad(G)$ by r and $diamG$ by d. Two vertices u and v are said to be antipodal $d(u, v) = d$. For two vertices u and v, the closed interval $I_e[u, v]$ consists of all edges lyingin a $u - v$ geodesic. If u and v are adjacent, then $I_e[u, v] = \{uv\}$. For a set S of
vertices, let $I_e[S] = \cup_{u,v \in S} I_e[u, v]$. A set $S \subseteq V$ is called an edge geodetic set of Gif $I_e[S] = E$. A set $S \subseteq V(G)$ is called an edge geodetic set of G if $I_e[S] = E$. Theedge geodetic number $g_e(G)$ of G is the minimum order of its edge geodetic sets andany edge geodetic set of order $g_e(G)$ is an edge geodetic basis or a g_e-set of G. An edge geodetic set S of G is said to be aconnected edge geodetic set of G if $G[S]$ is

2. NUMBER OF A GRAPH

Definition 2.1.A complement connected edge geodetic set of G is called a minimal complement connected edge geodetic set of G if no proper subset of S is a complement connected edge

e-ISSN 2320 –7876 www.ijfans.org
Vol.11, Iss.12, 2022
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geodetic set of G. The upper complement connected edge geodetic number $g^{+}_{cce}(G)$ is the maximum cardinality of a minimal complement connected edge geodetic set of G.

Remark 2.3. Every minimal complement connected edge geodetic set of G is a minimal complement connected edge geodetic set of G. But the converse need not be true. For the graph G given in Figure 2.1, $S_2 = \{v_2, v_3, v_5, v_7\}$ is a minimal complement connected edge geodetic set of G. But not a minimum complement connected edge geodetic set of G.

Observation 2.4. (i) Each semi-extreme vertex of a graph G belongs to every minimal complement connected edge geodetic set of G.

(ii) Let W be the set of all semi-extreme vertices of G. If W is the unique minimum complement connected edge geodetic set of G, $g^{+}_{cce}(G) = |\, W\,|$.

(iii) No cut vertex of a graph G belongs to any minimal complement connected edge geodetic set of G.

Observation 2.5. (i) For the tree T with k end vertices, $g^{+}_{cce}(G)$= k.

(ii) For the complete graph $G = K_p\ p \geq 2, g^{+}_{cce}(G)$ = p.

(iii) If G is a semi-complete graph, then $g^{+}_{cce}(G)$ = p.

(iv) For the wheel $W_p = K_1 + C_{p-1}\ p \geq 4, g^{+}_{cce}(G)$ = p – 1.

Theorem 2.6. For the cycle $G = C_p$,

$$g^{+}_{cce}(C_p) = = \begin{cases} \frac{p}{2} + 1, & if\ p\ is\ even \\ \frac{p+3}{2}, & if\ p\ is\ even \end{cases}$$

Proof: Let C_p be $v_1, v_2, \ldots, v_p, v_1$.

Case 1p be even. Let $p = 2n\ (n \geq 2)$. Let $S = \{v_1, v_2, \ldots, v_{n+1}\}$. Then $I_e[S] = E(G)$. $G[V - S]$ is connected. Therefore S is a complement connected edge geodetic set of G. We prove that S is a minimal complement connected edge geodetic set of G. On the contrary suppose that S is not a minimal complement connected edge geodetic set of G. Then there exists a complement connected edge geodetic set S_1 such that $S_1 \subset S$. Let x be a vertex of S such that $x \notin S_1$. If $x = v_1$ or v_{n+1}, $I_e[S_1] \neq E(G)$. If $x = v_i$ for $i\ (2 \leq i \leq n)$, then $G[V - S_1]$ is not connected. Therefore S_1 is not a complement connected edge geodetic set of G. Hence S_1 is a minimal complement connected edge geodetic set of G and so $g^{+}_{cce}(G) \geq n + 1$. We prove that $g^{+}_{cce}(G)$ = n. On the contrary that $g^{+}_{cce}(G) \geq m > n + 1$.

Case 2p is odd ,$(n \geq 3)$. Let $p = 2n + 1$. Let $S = \{v_1, v_2, \ldots, v_n, v_{n+1}, v_{n+2}\}$. Then as in Case 1, we can prove that $g^{+}_{cce}(G)$ = n + 2 =$\frac{p+3}{2}$.

II Some Result onUpper Complement Connected Edge Geodetic Number of a Graph

Observation 3.1. For a connected graph G of order $p \geq 2, 2 \leq g_{cce}(G) \leq g^{+}_{cce}(G) \leq p$

The following theorems shows that the bounds in Observation 3.1 can be sharp and strict.

Theorem 3.2. For a connected graph $G = P_{p_1} \times P_{p_2} (p_1, p_2 \geq 2), g^{+}_{cce}(G)$ = 2.

Proof: Let P_{p_1} denotes a path on $p_1$vertices and P_{p_1} denotes a path on p_2 vertices.For $p_1, p_2 \geq 2, P_{p_1} \times P_{p_2}$is defined as the two-dimensional mesh with $p_1$rows and$p_2$columns. It is denoted by $M_{p_1 \times p_2}$ for $1 \leq i \leq p_1$ and $1 \leq j \leq p_2$, we denote the ith row and jth column vertex of $M_{p_1 \times p_2}$as x_{ij}.

Theorem 3.3. For the complete bipartite graph $G = K_{m,n}, (2 \leq m \leq n), g_{cce}(G) = g^{+}_{cce}(G) =$

m + n − 1.

Proof: Let $X = \{x_1, x_2, .., x_m\}$, and $Y = \{y_1, y_2, .., y_n\}$ be the two bipartite of G. Let $S = V(G) - \{y_n\}$. Then S is a complement connected edge geodetic set of G and so $g_{cce}(G) \leq m + n - 1$. We prove that $g_{cce}(G) = m + n - 1$. On the contrary suppose that $g_{cce}(G) \leq m + n - 2$. Then there exists a complement connected edge geodetic set of S' such that $|S'| \leq m + n - 2$. Since $G[V - S']$ is connected, it follows that either$S' \subsetneq X$ or $S' \subsetneq Y$ or $S' \subset X \cup Y$. If $S' \subsetneq X$, then there exists $x \in X$ such that $x \notin S'$. Let e be an edge incident with x Then $e \notin I_e[S']$. Therefore S' is not a complement connected edge geodetic set of G. If $S' \subsetneq$ Y, then by the similar way, we prove that S' is not a complement connected edge geodetic set of G

Theorem 3.4. For the graph $G = K_p - \{e\}, p \geq 4$, Where e is an edge of K_p,$g^+_{cce}(G)$ = p.

Proof: Since G is a semi-complete graph, the result follows from Observation 2.5(iii).

Definition 3.5. Let C_6 be $v_1, v_2, v_3, v_4, v_5, v_6, v_1$. Let H be the graph obtained P_6 byintroducing a new vertex x and introducing the new edges xv_1, xv_3, xv_4 and xv_5. LetG_a be the graph obtained from H by introducing new vertices $z_1, z_2, \ldots, z_a$ by joiningeach z_i $(1 \leq i \leq a)$ with v_2 and v_6.

Theorem 3.6. For the graph $G = G_{a-5}(a \geq 7)$,
$g_{cce}(G) = 6$ and $g^+_{cce}(G)$ = a.

Proof: It can be easily verified that $S = \{v_1, v_2, v_3, v_4, v_5, x\}$ is a $g_{cce}(G) = 6$. Let $S_1 = \{x, v_1, v_2, v_3, v_4\} \cup \{z_1, z_2, \ldots, z_{a-5}\}$. Then S_1 is a complement connected edge geodetic set of G. We prove that S_1 is a minimal complement connected edge geodetic set of G. On the contrary suppose that S_1 is not a complement connected edge geodetic set of G. Then there exists a complement connected edge geodetic set S_2 such that $S_2 \subset S_1$. Let y be a vertex of S_1such that $y \notin S_2$. If $y = z_i$ $(1 \leq i \leq a - 5)$, then $v_2z_i, v_6z_i \notin I_e[S_2]$ for $(1 \leq i \leq a - 5)$. If $y = v_i$ $(1 \leq i \leq 4)$, then there exists at least one $e \in E(G)$ such that $e \notin I_e[S_2]$. Then there exists a complement connected edge geodetic set M of G such that $|M| \geq a + 1$. Since S, S_1 and S_3 are complement connected edge geodetic sets of $G, S \nsubseteq M, S_2 \nsubseteq M$ and $S_3 \nsubseteq M$. Also, since $p = a + 2$, we have $|M| = a + 1$. Since $G[V - M]$ is connected, either v_2 or $v_6 \in M$. We assume that $v_2 \in M$. Therefore $v_5 \in M$. Hence it follows that $S_3 \subset M$, which is a contradiction. Therefore $g^+_{cce}(G) = a$.

Observation 3.7. Let G be a connected graph with $\Delta(G) = p - 1$.

(i) If G contains only one universal vertex, then $g^+_{cce}(G) = p - 1$.

(ii) If G contains at least two universal vertices, then $g^+_{cce}(G)$= p.

Theorem 3.8. For a connected graph G of order , $g^+_{cce}(G) = p$ if and only if $g_{cce}(G) = p$.

Proof: Let $g^+_{cce}(G) = p$. Then $S = V(G)$ is the unique minimal complement connected edge geodetic set of G. Since no proper subset of S is a complement connected edge geodetic set, it is clear that S is the unique minimum complement connected edge geodetic set of G and so $g_{cce}(G) = p$. The converse follows from Theorem 3.1.

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