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On Some Sets in Digital Topology

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ABSTRACT: The notations of strong and week forms of open sets and closed sets in the digital line and digital plane have been used in digital image filtering techniques. The purpose of this paper is to characterize t\such sets in Digital Topology with special reference to α – closed and α- open sets.

Keywords—Digital Topology, Khalimsky topology, α- closed sets, α- open sets

I. INTRODUCTION

Digital topology deals with properties and features of two-dimensional (2D) or threedimensional (3D) digital images that correspond to topological properties (e.g., connectedness)
or topological features (e.g., boundaries) of objects. Concepts and results of digital topology are used to specify and justify important (low-level) image analysis algorithms, including algorithms for thinning, border or surface tracing, counting of components or tunnels, or region-filling. Digital topology was first studied in the late 1960s by the computer image analysis researcher Azriel Rosenfeld, whose publications on the subject played a major role in establishing and developing the field. The term" digital topology" was itself invented by Rosenfeld, who used it in a 1973 publication for the first time.

The information required for a digital picture can be stored by specifying the colour at each pixel. If a digital picture is formed by simple closed curve, one can specify the pixels on the simple closed curves and then specify uniformly the colours for the insides and the outside. This method results in the reduction of computer memory usage signicantly. This method employs the celebrated Jordan Curve Theorem, which states that every simple closed curve in the plane separates the plane into two connected components. Kong and Kopperman gave a topological approach to digital topology. Kong et al. studied the digital fundamental group and also established that on a strongly normal digital
picture space, the discrete and continuous concepts are equivalent. Maki et al. investigated the digital line and operations approaches of $T_{\frac{1}{2}}$spaces. Devi et al. studied the topologicalproperties of wgp-closed sets in the digital plane. Saha et al. investigated that the basic parts of digital geometry can be generalized into sets of convex voxels. Thangavelu discussed the properties of non-empty digital intervals [a; b] \Z and cardinalities of subspace topology on digital intervals are characterized. Throughout this paper, (X,), (Y, σ) and ($\mathbb{Z}$, κ) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), Cl(A), Int(A) and $X\backslash A$ denote the closure of A, the interior of A and the complement of A in X, respectively.

II. PRELIMINARIES

Definition: 2.1

A subset A of a space X is said to be
i) Regular open [21] if A = Int(Cl(A)
ii) Semi-open [15] if A ⊆Cl(Int(A))

iii) Preopen [17] if A ⊆ Int (Cl (A))
iv) Semi-preopen [2] or β-open [1] if A ⊆ Cl (Int (Cl (A)))
v) α -open [18] if A ⊆ Int (Cl (Int (A)))

Definition: 2.2
A subset A of a space X is said to be
i) p-set [23] if Cl (Int (A)) ⊆ Int (Cl (A))
ii) q-set [24] if Int (Cl (A)) ⊆ Cl (Int (A))
iii) t-set [25] if Int (A) = Int (Cl (A))
iv) t *-set [11] if Cl (A) = Cl (Int (A))

Lemma: 2.3
[12] Let A be a subset of $\mathbb{Z}$. Then
i) A is open if and only if for every $x \in A$, the following hold:
(x is odd) or (x is even with $x-1, x+1 \in A$).
ii) A is closed if and only if for every $x \in A$, the following holds: (x is even) or (x is odd with $x-1, x+1 \in A$).

Let $\aleph(x)$denote the smallest neighbourhood of $x$in $(\mathbb{Z},\kappa)$ Then

$$\aleph(x) = \begin{cases} \{x\} & If \;\; x \; is \; odd \\ \{x-1, x, x+1\} & If \; x \; is \; even \end{cases}$$

Let $\mathbb{C}(x)$denote the smallest closed set containing $x$in $(\mathbb{Z},\kappa)$ Then

$$\mathbb{C}(x) = \begin{cases} \{x\} & If \;\; x \; is \; even \\ \{x-1, x, x+1\} & If \; x \; is \; odd \end{cases}$$

The topology satisfying the properties in Lemma 2.3 is exactly the Khalimsky line topology on $\mathbb{Z}$. Based on the concepts of "open" and "closed" in Lemma 2.3, we obtain the topology on $\mathbb{Z}$, which is referred to as the Khalimsky line topology on Z.

III. CHARACTERIZATION OF SETS IN KHALIMSKY TOPOLOGY

Proposition 2.1
If x is an odd integer, then the set $\{x\}$ is regular open, semi-open, semi-preopen, a q-set and a t*-set in $(\mathbb{Z},\kappa)$.
Proof:
Suppose x is odd.Then Int ($\{x\}$ = $\{x]$ and Cl (Int ($\{x\}$)) = $\{x-1, x, x+1\}$. Also Cl ($\{x\}$) = $\{x-1, x, x+1\}$ and
Int (Cl ($\{x\}$)) = $\{x]$. By Definition 2.1, Since $\{x\}$ = Int(Cl($\{x\}$) by Definition 2.1 (i), $\{x\}$is regular open and $\{x\}$ ⊆Cl(Int($\{x\}$= $\{x-1, x, x+1\}$ by Definition 2.1 (ii), $\{x\}$ is semi-open. Also, $\{x\}$ ⊆ Cl (Int (Cl ($\{x\}$)

by Definition 2.1 (iv), $\{x\}$ is semi-preopen and Int (Cl ($\{x\}$))⊆ Cl (Int ($\{x\}$)) by Definition 2.2 (ii), $\{x\}$ is a q-set. The relation Cl ($\{x\}$) = Cl (Int ({x})) implies that $\{x\}$ is a t^*-set.

Proposition 2.2
If $\{x\}$ is an even integer, the set $\{x\}$ is α-open, α-closed, preclosed, semi-closed, t-set, t^*-set in $(\mathbb{Z}, \kappa)$.
Proof:
Let $\{x\}$ be even. Observe that Int ({x}) = ∅ and Cl (Int ($\{x\}$)) = ∅. Then Cl ($\{x\}$) = ∅ and Int (Cl ($\{x\}$) = ∅. So Int (Cl (Int ($\{x\}$) = $\{x-1, x, x+1\}$ and Cl (Int (Cl ($\{x\}$))) = ∅.

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