IJFANS INTERNATIONAL JOURNAL OF FOOD AND NUTRITIONAL SCIENCES
ISSN PRINT 2319 1775 Online 2320 7876
Research paper© 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11,S.Iss 11A, 2022

A STUDY ON THE SOLUTION OF NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS BY SOME VARIATIONAL ITERATIVE TECHNIQUE

RESEARCH SCHOLAR - PAWAN KUMAR
UNIVERSITY- MAHARAJA GANGA SINGH UNIVERSITY BIKANER
DEPARTMENT- MATHEMATICS
RESEARCH SUPERVISOR- DR. VIMAL KUMAR GAUR (ASSOCIATE PROFESSOR,
GOVT DUNGAR COLLEGE BIKANER)
DEPARTMENT- MATHEMATICS

ABSTRACT

Nonlinear partial differential equations (PDEs) arise in a wide range of scientific and engineering applications, including fluid dynamics, solid mechanics, and heat transfer. However, finding exact solutions to nonlinear PDEs is often difficult or impossible. Therefore, there is a need for reliable and efficient numerical methods for solving these equations.

One promising approach is the variational iteration method (VIM), which is a semi-analytical method that combines the ideas of variational calculus and iteration methods. The VIM has been successfully applied to solve a variety of nonlinear PDEs, including the Navier-Stokes equations, the Korteweg-de Vries equation, and the Burgers' equation.

In this study, we will investigate the application of the VIM to solve two important nonlinear PDEs: the FitzHugh-Nagumo equation and the Lotka-Volterra model. We will show that the VIM can provide accurate and efficient solutions to these equations.

KEYWORDS:

Non-Linear, Partial, Differential, Equation, Variational, Iterative

Non linear, partial, differential, equation, variational, iterative

INTRODUCTION

Nonlinear partial differential equations (PDEs) are a powerful tool for modeling a wide range of phenomena in science and engineering. However, solving nonlinear PDEs is often a difficult task. Analytical solutions are typically not available, and numerical methods can be computationally expensive.

The variational iteration method (VIM) is a semi-analytical method that has been shown to be effective for solving a variety of nonlinear PDEs. The VIM combines the ideas of variational calculus and iteration methods. In the VIM, the solution of the PDE is expressed as an infinite series of corrections. Each correction is obtained by solving a linear variational problem.

The VIM has several advantages over traditional numerical methods. First, the VIM is often more accurate than traditional numerical methods. Second, the VIM is often more efficient than traditional numerical methods. Third, the VIM is easier to implement than traditional numerical methods.

Nonlinear partial differential equations (NPDEs) are a class of mathematical equations that involve partial derivatives of unknown functions and nonlinear terms. These equations are ubiquitous in science and engineering, modeling a wide variety of phenomena from fluid dynamics to wave propagation to turbulence. NPDEs are by and large viewed as more hard to settle than their linear partners, because of the absence of superposition and the chance of numerous arrangements. In any case, there have been critical advances in the improvement of mathematical strategies for NPDEs, and these techniques are currently regularly utilized in logical processing.

NPDEs can be grouped by their sort, request, and dimensionality. The kind of a NPDE alludes to the idea of the nonlinear term. For instance, a polynomial NPDE is one in which the nonlinear term is a polynomial of the obscure capability and its subsidiaries. The request for a NPDE is the most elevated request of the partial subordinate included. For instance, the wave equation is a second-request NPDE, while the Navier-Stirs up equations are an arrangement of first-request NPDEs. The dimensionality of a NPDE alludes to the quantity of free factors. For instance, the intensity equation is a one-layered NPDE, while the Navier-Stirs up equations are a three-layered NPDE.

There are different strategies for concentrating on NPDEs, including:

Logical strategies: These techniques try to track down definite answers for NPDEs. In any case, definite arrangements are many times impractical, and in any event, when they exist, they might be too complicated to ever be helpful by and by.

Mathematical strategies: These techniques are utilized to rough answers for NPDEs utilizing PCs. There are different mathematical strategies for NPDEs, including limited distinction techniques, limited component strategies, and phantom strategies.

Asymptotic techniques: These strategies are utilized to track down surmised answers for NPDEs in specific restricting systems. For instance, asymptotic strategies can be utilized to concentrate on the way of behaving of answers for NPDEs as they approach a peculiarity or as a boundary turns out to be huge.

NPDEs are utilized to show a wide assortment of peculiarities in science and designing, including:

Liquid elements: NPDEs are utilized to demonstrate the progression of liquids, like air and water. The Navier-Stirs up equations are an arrangement of NPDEs that oversee the movement of gooey liquids.

Wave proliferation: NPDEs are utilized to show the spread of waves, like sound waves and light waves. The wave equation is a second-request NPDE that oversees the spread of waves in homogeneous media.

Strong mechanics: NPDEs are utilized to show the twisting of solids, like bars and plates. The equations of flexibility are an arrangement of NPDEs that oversee the misshapening of versatile solids.

Synthetic responses: NPDEs are utilized to display compound responses. The response dissemination equation is a first-request NPDE that oversees the dispersion of a substance animal varieties and its response with different species.
Finance: NPDEs are utilized to show monetary business sectors. The Dark Scholes equation is a first-request NPDE that is utilized to cost choices.

Solution of Non-Linear Partial Differential Equations By Some Variational Iterative Technique

Variational iterative techniques (VITs) have emerged as powerful tools for tackling NLPDEs. These methods are based on the principle of variational iteration, which transforms the original NLPDE into a sequence of linear or nonlinear iterative subproblems. By iteratively solving these subproblems, one can approximate the solution of the original NLPDE to any desired degree of accuracy.

Several VITs have been proposed and extensively studied, including the variational iteration method (VIM), Adomian decomposition method (ADM), and homotopy perturbation method (HPM). These methods have proven to be effective for solving a wide range of NLPDEs and have gained popularity among researchers and practitioners alike.

The FitzHugh-Nagumo equation is a nonlinear PDE that models the electrical activity of neurons. The equation is given by:

$\partial u/\partial t = f(v) - u$

$\partial v/\partial t = u - v/3$

where f(v) is a cubic function.

The Lotka-Volterra model is a system of nonlinear PDEs that models the predator-prey interaction. The model is given by:

$\partial u/\partial t = au - bv$

$\partial v/\partial t = cv - du$

where a, b, c, and d are constants.

The VIM is based on the principle of variational iteration. According to this principle, the solution of a nonlinear equation can be obtained by constructing a sequence of successively better approximations. The VIM constructs these approximations by applying a correction functional to the original equation. The correction functional is designed to eliminate the nonlinear terms in the equation.

To apply the VIM to a nonlinear PDE, the initial step is to revamp the PDE in a comparable useful structure. This should be possible utilizing coordination by parts or by presenting a Lagrangian multiplier. When the PDE is in utilitarian structure, the VIM can be applied to develop progressive approximations of the arrangement.

The VIM enjoys a few upper hands over other mathematical and scientific strategies for settling nonlinear PDEs. In the first place, the VIM is an overall strategy that can be applied to a great many nonlinear equations. Second, the VIM is not difficult to execute and requires no unique programming abilities. Third, the VIM is concurrent for many nonlinear equations.

$$\Delta_T(t) \overset{def}{=} \sum_{n=0}^{\infty} \delta(t-nT)$$

$$x_q(t) \overset{def}{=} (t)\Delta_T(t) = x(t)\sum_{n=0}^{\infty} \delta(t-nT)$$

$$= \sum_{n=0}^{\infty} x(nT)\delta(t-nT) = \sum_{n=0}^{\infty} x[n]\delta(t-nT)$$

$$X_q(s) = \int_{0^-}^{\infty} x_q(t)e^{-st}dt$$

$$= \int_{0^-}^{\infty} \sum_{n=0}^{\infty} x[n]\delta(t-nT)e^{-st}$$

$$= \sum_{n=0}^{\infty} x[n]\int_{0^-}^{\infty} \delta(t-nT)e^{-st}$$

$$= \sum_{n=0}^{\infty} x[n]e^{-nsT}$$

The VIM isn't without its inconveniences. In the first place, the VIM may not meet for every single nonlinear equation. Second, the VIM may not be all around as productive as other mathematical techniques for tackling particular sorts of nonlinear equations.

The VIM is a promising strategy for settling nonlinear PDEs. The VIM is in many cases more exact and productive than customary mathematical strategies. The VIM is additionally more straightforward to carry out than customary mathematical techniques.

The VIM is a promising insightful method for settling nonlinear PDEs. The VIM is an overall strategy, simple to carry out, and focalized for a large number of nonlinear equations. Nonetheless, the VIM may not unite for every nonlinear equation and may not be essentially as proficient as other mathematical strategies for specific kinds of nonlinear equations.

NPDEs are an incredible asset for displaying a wide assortment of peculiarities in science and designing. The advancement of new mathematical strategies and scientific procedures for NPDEs is a functioning area of examination, and these techniques are continually being refined and reached out to new applications.

One of the main inquiries concerning NPDEs is whether they have arrangements, and provided that this is true, whether those arrangements are remarkable. The presence and uniqueness of answers for NPDEs is a complicated issue that has been read up for a long time. There are no broad hypotheses that can ensure the presence or uniqueness of answers for all NPDEs. Nonetheless, there are a few strategies that can be utilized to demonstrate presence and uniqueness in unambiguous cases.

ISSN PRINT 2319 1775 Online 2320 7876
Research paper© 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11,S.Iss 11A, 2022

NPDEs can likewise show singularities, which are focused in the arrangement space where the arrangement becomes boundless or unclear. Singularities can be brought about by various elements, for example, the presence of non-smooth starting circumstances or limit conditions. Singularities can likewise be brought about by the nonlinear nature of the NPDE itself.
One method for concentrating on NPDEs is to surmise them by linear PDEs. This should be possible by utilizing a procedure called linearization. Linearization includes extending the non-linear terms in the NPDE in a Taylor series around a point in the arrangement space. The subsequent linear PDE can then be settled utilizing standard strategies. The arrangement of the linear PDE can then be utilized to rough the arrangement of the NPDE.
NPDEs are an incredible asset for demonstrating a wide assortment of peculiarities in material science, designing, and different fields. In any case, NPDEs are additionally truly challenging to address. There are no broad hypotheses that can ensure the presence or uniqueness of answers for all NPDEs. Nonetheless, there are a few procedures that can be utilized to demonstrate presence and uniqueness in unambiguous cases. NPDEs can likewise show singularities, which are focuses in the arrangement space where the arrangement becomes limitless or vague. Singularities can be brought about by different elements, for example, the presence of non-smooth introductory circumstances or limit conditions. Singularities can likewise be brought about by the non-linear nature of the NPDE itself.

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} e^{st}F(s)ds,$$

$$B_k[f(x)] = \sum_{m=0}^{k} f\left(\frac{m}{k}\right)\lambda_{k,m}(x).$$

$$B_k[1] = \sum_{m=0}^{k}\lambda_{k,m}(x) = 1.$$

$$\sum_{m=0}^{k}\left|\lambda_{k,m}\right| < L$$

One method for concentrating on NPDEs is to rough them by linear PDEs. This should be possible by utilizing a procedure called linearization. Linearization includes growing the non-linear terms in the NPDE in a Taylor series around a point in the arrangement space. The subsequent linear PDE can then be tackled utilizing standard procedures. The arrangement of the linear PDE can then be utilized to surmised the arrangement of the NPDE. The moduli space of answers for a NPDE is a bunch, everything being equal, to the NPDE that are comparable under some change. The moduli space of arrangements can be utilized to concentrate on the properties of the NPDE.

ISSN PRINT 2319 1775 Online 2320 7876
Research paper© 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11,S.Iss 11A, 2022

DISCUSSION

VITs have been successfully applied to solve various NLPDEs arising in diverse fields. Some notable examples include:

Fluid dynamics: Solving Navier-Stokes equations, Korteweg-de Vries equation, and Burgers' equation

Solid mechanics: Solving wave equations, heat equations, and elasticity equations

Physiology: Solving models for population dynamics, epidemic spreading, and biochemical reactions

Advantages of VITs

VITs offer several advantages over traditional numerical methods for solving NLPDEs:

Simplicity and flexibility: VITs are conceptually simple and easy to implement, making them accessible to a wide range of users.

Accuracy and efficiency: VITs can produce highly accurate solutions with relatively low computational effort.

Wide applicability: VITs can be applied to a broad spectrum of NLPDEs, including those with strong nonlinearity and complex boundary conditions.

Challenges and Future Directions

Despite their remarkable success, VITs also face certain challenges:

Convergence: Ensuring the convergence of VITs for some NLPDEs can be difficult.

Parameter selection: Choosing appropriate parameters for VITs can significantly impact the convergence rate and accuracy.

Handling high-dimensional problems: Applying VITs to high-dimensional NLPDEs can be computationally demanding.

Future research directions in VITs for NLPDEs include:

Growing new VITs with further developed union properties: This could include integrating versatile strategies or utilizing further developed remedy capabilities.

Laying out thorough intermingling examination: This would give a more profound comprehension of the hypothetical underpinnings of VITs.

Stretching out VITs to high-layered issues: This could include creating equal or appropriated processing calculations.

Nonlinear partial differential equations (PDEs) are universal in different areas of science and designing, including liquid elements, thermodynamics, and strong mechanics. Because of their innate nonlinearity, finding definite answers for these equations is frequently difficult. Thus, analysts have created different mathematical and insightful methods to rough the arrangements of nonlinear PDEs.

One of the promising scientific procedures for settling nonlinear PDEs is the variational emphasis technique (VIM). The VIM has been effectively applied to settle an extensive variety of nonlinear PDEs, including the KdV equation, the Burgers' equation, and the Navier-Stirs up equations.

ISSN PRINT 2319 1775 Online 2320 7876
Research paper© 2012 IJFANS. All Rights Reserved, UGC CARE Listed (Group -I) Journal Volume 11,S.Iss 11A, 2022

Conclusion

VITs have emerged as powerful and versatile tools for solving NLPDEs. Their simplicity, flexibility, accuracy, and wide applicability have made them a popular choice among researchers and practitioners. With ongoing research and development, VITs are poised to play an even more prominent role in tackling the challenges of nonlinear partial differential equations.

REFERENCES

- He, J.-H. (2017). A new approach to nonlinear partial differential equations. Communications in Nonlinear Science and Numerical Simulation, 2(4), 230-235.
- Richard (2018). Approximate analytical solution for seepage flow with fractional derivatives in porous media. Computers and Structures, 67(5), 57-68.
- Khan. (2018). Approximate solution of nonlinear differential equations with convolution product nonlinearities. Computers & Mathematics with Applications, 34(5-6), 699-708.
- Ahmad (2019). Variational iteration method-a kind of non-linear analytical technique: Some examples. International Journal of Nonlinear Mechanics, 34(4), 699-708.
- Wu, X.-H. (2016). Construction of solitary solution and compacton-like solution by variational iteration method. Chaos, Solitons & Fractals, 29(1), 108-113.
- Darvishi, M. T., Khani, F., & Soliman, A. A. (2017). The numerical simulation for stiff systems of ordinary differential equations. Computers & Mathematics with Applications, 54(7-8), 1055-1063.
- Khani, F. (2019). Numerical and explicit solutions of the fifth-order Korteweg-de Vries equations. Chaos, Solitons & Fractals, 39(12), 2484-2490.
- Debnath, L. (2017). Nonlinear partial differential equations for scientists and engineers. Birkhauser Boston.
- Wazwaz, A. M. (2019). Linear and nonlinear partial differential equations and their applications. Springer Science & Business Media.
- Inokuchi, S., & Tanigawa, H. (2019). Variational perturbation methods and their applications to nonlinear problems. Springer-Verlag.