

## Exploring Magnetohydrodynamic (MHD) Radiative Flow of a Casson Fluid Through a Forchheimer Permeable Medium with Consideration for Joule Heating Effects.

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### Abstract

The present investigation focuses on the numerical analysis of Casson fluid flow over an inclined nonlinear surface through a Darcy-Forchheimer permeable medium. This study takes into account various influential factors, including viscous dissipation, magnetohydrodynamics (MHD), radiation, chemical reaction, and Joule heating effects. The governing partial differential equations (PDEs) governing the problem are transformed into nonlinear ordinary differential equations (ODEs) using appropriate similarity transformations. The ODEs are subsequently solved utilizing the Keller Box method. The impact of the involved parameters is assessed by constructing profiles for velocity, temperature, and concentration. The velocity profile demonstrates a decrease with increasing values of the Casson parameter and the magnetic parameter, while it increases with higher permeability parameter values. Temperature exhibits an increase with progressive radiation parameter, Dufour parameter values, and the concentration profile experiences a decrease with higher values of the chemical reaction parameter.

### Introduction

Fluid flow influenced by Magnetohydrodynamics (MHD) finds utility across diverse industrial applications, including MHD generators and the design of nuclear reactors. Additionally, MHD theory aids in comprehending phenomena observed in celestial bodies within the solar system and even in distant astrophysical realms [5-7]. MHD constitutes the scientific exploration of the behavior of electrically conductive fluids in the presence of a magnetic field. This interaction involves inducing currents within the moving conductive fluid's magnetic field, leading to the polarization of the liquid and a reversal of the magnetic field direction. Mathematically, the principles of MHD are described through the amalgamation of the Navier-Stokes equations of fluid dynamics with Maxwell's equations of electromagnetism. The dynamics of fluid flow are governed by adjustments in the observed electrical conductivities and viscosities of the fluid, as outlined in the investigation conducted by Malashetty et al. [1], which pertained to MHD two-phase flow over an inclined channel. The application of a magnetic field is associated with a reduction in the temperature and concentration gradients at the wall, as noted in the study by Chen [2]. Furthermore, Alam et al. [3] employed an RK

integration scheme to explore the influence of MHD and heat sources on fluid flow over a slanted surface, with an emphasis on the augmentation of the magnetic field [4]. The current investigation examines the flow of a Casson fluid over an inclined extended surface within a Forchheimer permeable medium, taking into account the influences of a magnetic field, radiation, chemical reaction, viscous dissipation, slip effects, and Joule heating [8]. The corresponding equations are converted into nonlinear ordinary differential equations are further solved using the Keller box scheme [9].

## Mathematical formulation

We analyze the steady flow of a two-dimensional Casson fluid over a nonlinear extended surface within Forchheimer porous media. The surface is inclined at an angle  $\alpha$  in the vertical direction, with the stretching occurring along the x-axis [10-12]. The y-axis is perpendicular to the surface, and a uniform magnetic field is applied, as illustrated in Figure 1.

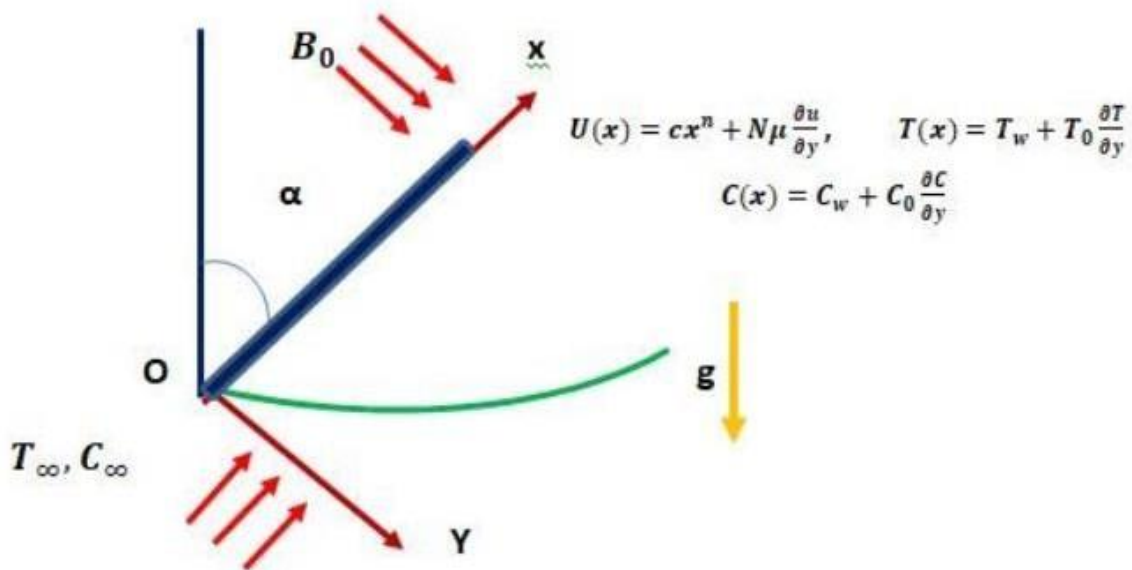


Fig 1: Flow model

$$u_x + v_y = 0 \quad (1)$$

$$u(u_x) + v(u_y) = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k_1} u - \frac{C_b}{\sqrt{k_1}} u^2 \pm g [\beta_T (T - T_\infty) + \beta_c (C - C_\infty)] \cos \alpha \quad (2)$$

$$u(T_x) + v(T_y) = \alpha(T_{yy}) + \frac{D_m k_t}{C_s C_p} (C_{yy}) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 \left( 1 + \frac{1}{\beta} \right) + \frac{\sigma B_0^2}{\rho C_p} u^2 \quad (3)$$

$$u(C_x) + v(C_y) = D_B(C_{yy}) + \frac{D_T}{T_\infty} (T_{yy}) - k_2(C - C_\infty) \quad (4)$$

Corresponding boundary conditions are

$$\text{At } y=0, \quad u = U(x) = cx^n + N\mu \frac{\partial u}{\partial y}, \quad v = -V(x), \quad T = T_w + N\mu \frac{\partial u}{\partial y} \quad (5)$$

$$\text{As } y \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty$$

Similarity transformations are

$$u = cx^n f'(\eta), \quad v = -\sqrt{\frac{c\nu(n+1)}{2}} x^{\frac{n-1}{2}} \left[ f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right] \quad (6)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}$$

Employing similarity transformations Eq. (6), Eq. (2), Eq. (3), Eq. (4) are transformed to

$$\left( 1 + \frac{1}{\beta} \right) f''' + ff'' - \frac{2N}{N+1} f'^2 - \frac{2}{N+1} \left[ \left[ M + \frac{1}{k_1} \right] f' - Frf'^2 + C_1 \theta \cos \alpha + C_2 \phi \cos \alpha \right] = 0 \quad (7)$$

$$\left( 1 + \frac{4}{3} Rd \right) \theta'' + Pr f \theta' + Pr Du \phi'' + Pr Ec \left( 1 + \frac{1}{\beta} \right) f'^2 + \frac{2}{N+1} Pr Jf'^2 = 0 \quad (8)$$

$$\phi'' + Scf \phi' + \frac{Nt}{Nb} \theta'' - \frac{2}{N+1} Sc\gamma \phi = 0 \quad (9)$$

Boundary conditions are transformed to

$$f(0) = S, \quad f'(0) = 1 + \lambda_u f''(0), \quad \theta(0) = 1 + \lambda_t \theta'(0), \quad \phi(0) = 1 + \lambda_c \phi'(0) \quad (10)$$

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0$$

## Numerical methodology

The implicit finite difference scheme called the Keller Box method is utilised to convert Eq (7)-(9) into ordinary differential equations of the first order [13].

$$\text{Introducing, } \frac{df}{d\eta} = p, \frac{dp}{d\eta} = q, g = \theta, \frac{dg}{d\eta} = t, s = \phi, \frac{ds}{d\eta} = n, \quad (11)$$

Eq. (7), Eq. (8) and Eq. (9) transformed to

$$\left(1 + \frac{1}{\beta}\right) q' + fq - \frac{2N}{N+1} p^2 - \frac{2}{N+1} \left(M + \frac{1}{k_1}\right) p - \frac{2}{N+1} Fr p^2 + \frac{2}{N+1} C_1 g \cos \alpha + \frac{2}{N+1} C_2 \cos \alpha = 0 \quad (12)$$

$$\left(1 + \frac{4}{3} Rd\right) t' + Pr ft + Pr Dun' + Pr Ec \left(1 + \frac{1}{\beta}\right) q^2 + \frac{2}{N+1} Pr Jp^2 = 0 \quad (13)$$

$$n' + Scfn + \frac{Nt}{Nb} t' - \frac{2}{N+1} Sc\gamma s = 0 \quad (14)$$

By using finite differences, and Newton's method, Eqs. (11) - (14) reduces to

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2} (\delta p_j + \delta p_{j-1}) = (r_1)_j \quad (15)$$

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2} (\delta q_j + \delta q_{j-1}) = (r_2)_j \quad (16)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2} (\delta t_j + \delta t_{j-1}) = (r_3)_j \quad (17)$$

$$\delta s_j - \delta s_{j-1} - \frac{h_j}{2} (\delta n_j + \delta n_{j-1}) = (r_4)_j \quad (18)$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + (a_6)_j \delta p_{j-1} \\ + (a_7)_j \delta g_j + (a_8)_j \delta g_{j-1} + (a_9)_j \delta s_j + (a_{10})_j \delta s_{j-1} = (r_5)_j \quad (19)$$

$$(b_1)_j \delta t_j + (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta n_j + (b_6)_j \delta n_{j-1} \\ + (b_7)_j \delta q_j + (b_8)_j \delta q_{j-1} + (b_9)_j \delta p_j + (b_{10})_j \delta p_{j-1} = (r_6)_j \quad (20)$$

$$(c_1)_j \delta n_j + (c_2)_j \delta n_{j-1} + (c_3)_j \delta f_j + (c_4)_j \delta f_{j-1} + (c_5)_j \delta t_j + (c_6)_j \delta t_{j-1} \\ + (c_7)_j \delta s_j + (c_8)_j \delta s_{j-1} = (r_7)_j \quad (21)$$

## Results

The effects of various profiles are analysed by developing velocity, temperature, and concentration profiles using the software MATLAB.

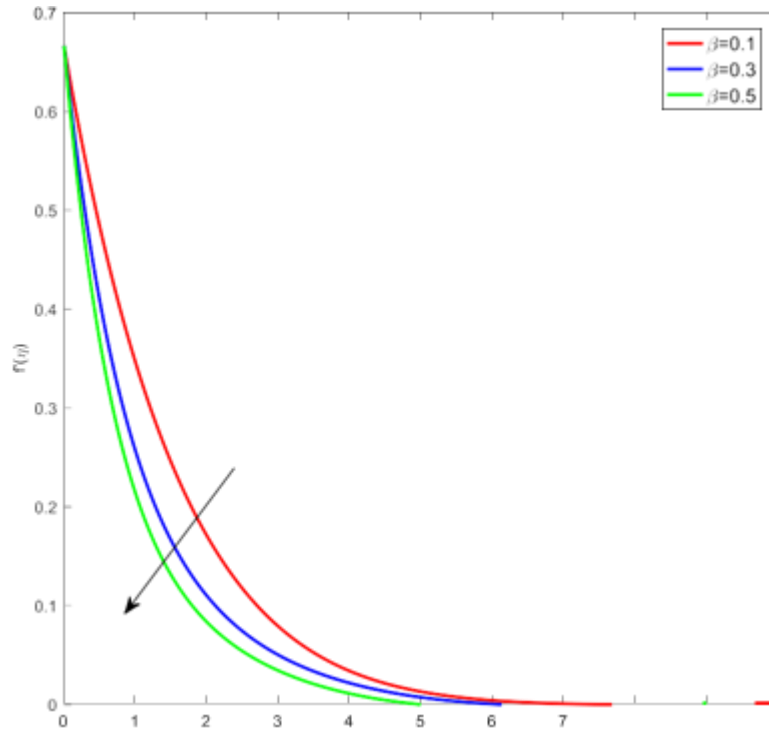


Fig. 2. Velocity graphs of Casson Parameter

Figure 3 depicts velocity profiles of Magnetic parameter. For enhanced observations of Magnetic parameter decrement in velocity profiles is noted.

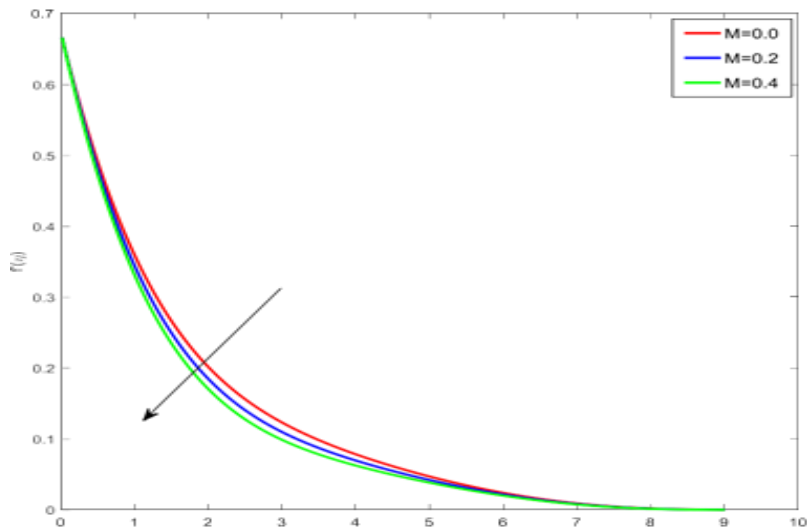


Fig. 3. Velocity profiles of Magnetic parameter

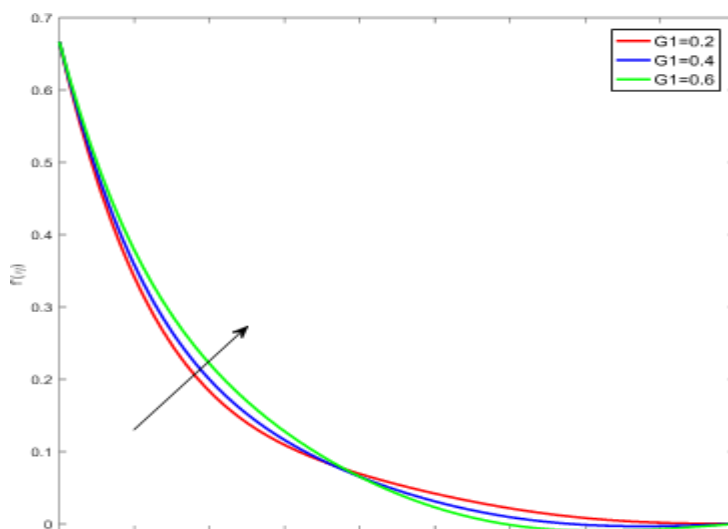


Fig 3: velocity graphs of magnetic parameter and buoyancy parameter

## Conclusions

In this study, we investigate the two-dimensional flow of a Casson fluid over a slanted nonlinear surface through a permeable medium characterized by the Forchheimer model. The analysis takes into account various influencing factors, including magnetohydrodynamics (MHD), thermal radiation, Joule heating, viscous dissipation, and chemical reaction. To solve the equations governing this complex problem, we employ the Keller Box method. The obtained numerical results lead to the following observations:

1. Velocity profiles decrease with increasing values of the Casson parameter, magnetic parameter, angle of inclination, Forchheimer number, and increase with buoyancy and permeability parameters.
2. Temperature profiles increase with higher values of the Dufour parameter, Eckert number, Joule heating, and radiation parameters. Conversely, they exhibit a decrease with the Prandtl number.
3. Concentration profiles rise with increased values of the thermophoresis parameter, while they decrease with higher Schmidt number, chemical reaction, and Brownian diffusion constraints.

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