LINEAR STABILITY ANALYSIS OF MAXWELL FLUID SATURATED POROUS LAYER

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Abstract
Linear stability of a fluid saturated porous medium heated from below is studied when the fluid and solid phases are not in local thermal equilibrium. The Darcy model which includes Coriolis term and permeability is employed as a momentum equation. The critical Rayleigh number for the onset of convection using linear stability analysis is found numerically as a function interphase heat transfer coefficient, aspect ratio.The effect of porosity modified conductivity ratio, diffusivity ratio, interphase heat transfer coefficient on the stability of the system is investigated.

8.1 Introduction

Basic understanding of natural convection in a non-Newtonian fluid-saturated porous medium is of interest in many engineering applications such as material processing, petroleum, chemical and nuclear industries, geophysics, bioengineering and reservoir engineering. The performance of a reservoir depends to a large extent upon the physical nature of crude oil present in the reservoir. The light crude is essentially Newtonian and is studied extensively using the Darcy equation. On the other hand, the heavy crude is non-Newtonian and a study of such fluids is based on a generalized Darcy equation, which takes into account the non-Newtonian effects. Such an equation is useful in the study of mobility control in oil displacement mechanism, which improves the efficiency of the oil recovery. Furthermore, some oil sand contains waxy crude at shallow depths of the reservoirs, which are considered to be viscoelastic fluids. In such situations, a viscoelastic model of a fluid will be more realistic than the Newtonian model. Besides, viscoelastic models are interesting because they fit quite well the data found in experiments of many polymeric fluids.

Since elastic behaviour is inherent in non-Newtonian fluids, it may be expected that oscillatory convection will sets up in such fluids. However, little if any attention was paid to the onset of oscillatory convection, which is the most dangerous mode for viscoelastic fluids. The corresponding problem in the case of a porous medium has also not received much attention.

Mathematical Formulation

We consider a sparsely packed porous layer of depth *d*, saturated with a viscoelastic fluid describable by the Maxwell model, which is heated from below and cooled from above. The lower surface is held at a temperature T_l, while the upper surface is at T_u $(T_l > T_u)$. We assume that the solid and fluid phases of the medium are not in local thermal equilibrium and use a two-field model for temperatures.

The basic equations governing the infinitesimal perturbations in the form,

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}-\rho' g\mathbf{k}+\nabla p'+\frac{\mu_f}{K}\mathbf{q}\right)-\mu_e\nabla^2\mathbf{q}=0\,, \tag{1}$$

$$\varepsilon(\rho_0 c)_f\frac{\partial T_f'}{\partial t}+(\rho_0 c)_f w'\left(\frac{dT_{fb}}{dz}\right)=\varepsilon k_f\nabla^2 T_f'+h\left(T_s'-T_f'\right), \tag{2}$$

$$(1-\varepsilon)(\rho_0 c)_s\frac{\partial T_s'}{\partial t}=(1-\varepsilon)k_s\nabla^2 T_s'-h\left(T_s'-T_f'\right), \tag{3}$$

$$\rho'=-\rho_0\beta T_f'\,, \tag{4}$$

$$\left(x^*,\ y^*,\ z^*\right)=\frac{(x,\ y,\ z)}{d},\quad t^*=\frac{k_f}{(\rho_0 c)_f d^2}t,\quad w^*=\frac{(\rho_0 c)_f d}{\varepsilon k_f}w'$$
$$T_f^*=\frac{T_f'}{(T_l-T_u)},\quad T_s^*=\frac{T_s'}{(T_l-T_u)}, \tag{5}$$

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left(1+\Gamma\frac{\partial}{\partial t}\right)\left[\frac{1}{\Pr}\frac{\partial}{\partial t}\left(\nabla^2 w\right)-R\nabla_1^2 T_f+\nabla^2 w\right]-Da\,\nabla^4 w=0\,, \tag{6}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T_f=w+H\left(T_s-T_f\right), \tag{7}$$

$$\left(\alpha\frac{\partial}{\partial t}-\nabla^2\right)T_s=\gamma H\left(T_f-T_s\right), \tag{8}$$

where

$\Gamma=\dfrac{\lambda_1 k_f}{(\rho_0 c)_f d^2}=\dfrac{\lambda_1\kappa_f}{d^2}$, the stress relaxation parameter,

$\Pr=\dfrac{\mu_f(\rho_c)_f d^2}{\varepsilon\rho_0 k_f K}=\dfrac{\upsilon d^2}{\varepsilon\kappa_f K}$, the Prandtl number ,

$Ra=\dfrac{\beta g d^3\,(T_l-T_u)(\rho c)_f}{\nu_f\kappa_f}$, the Rayleigh number,

$Da=\dfrac{\mu_e K}{\mu_f d^2}$, modified Darcy number,

$H=\dfrac{hd^2}{\varepsilon k_f}$, the inter-phase heat transfer coefficient,

$\alpha=\dfrac{(\rho_0 c)_s k_f}{(\rho_0 c)_f k_s}=\dfrac{\kappa_f}{\kappa_s}$, the ratio of diffusivities,

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$\gamma = \dfrac{\varepsilon k_f}{(1-\varepsilon) k_s}$, the porosity modified conductivity ratio.

Since the fluid and solid phases are not in thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption that the solid and fluid phases have equal temperatures at the bounding surfaces made earlier helps in overcoming this difficulty. Accordingly, Equations (6)-(8) are solved for impermeable isothermal boundaries. Hence the boundary conditions are

$$w = \frac{d^2 w}{dz^2} = 0 \quad \text{at} \quad z = 0 \text{ and } 1, \tag{9}$$

$$T_f = T_s = 0 \quad \text{at} \quad z = 0 \text{ and } 1. \tag{10}$$

Linear stability analysis

We assume the normal mode solutions for the eigen value problem defined by Equations (5)-(8) subject to the boundary conditions (9) and (10) in the form

$$\begin{pmatrix} w \\ T_f \\ T_s \end{pmatrix} = \begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} \exp\{i(lx + my) + \omega t\}, \tag{11}$$

where l and m are the horizontal wave numbers and ω is the growth rate. Substituting Equations (11) into Equations (9)-(10) we obtain

$$(1+\Gamma\omega)\left[\frac{\omega}{\Pr}\left(D^2 - a^2\right)W + Raa^2\Theta + \left(D^2 - a^2\right)W\right] - Da\left(D^2 - a^2\right)^2 W = 0, \tag{12}$$

$$\left[\left(D^2 - a^2\right) - \omega\right]\Theta + W + H(\Phi - \Theta) = 0, \tag{13}$$

$$\left[\left(D^2 - a^2\right) - \alpha\omega\right]\Phi + \gamma H(\Theta - \Phi) = 0, \tag{14}$$

where $D = \dfrac{d}{dz}$ *and* $a^2 = l^2 + m^2$.

The boundary conditions now become,

$$W = D^2 W = \Theta = \Phi = 0, \quad \text{at} \quad z = 0, \text{ and } 1. \tag{15}$$

We assume the solution to W, Θ and Φ in the form,

$$\begin{pmatrix} W \\ \Theta \\ \Phi \end{pmatrix} = \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \sin \pi z \tag{16}$$

which satisfy the boundary conditions (15). Substituting Equation (16) we obtain the following matrix equation

$$\begin{pmatrix} \delta^2\left\{1+\frac{\omega}{\Pr}+\frac{Da\,\delta^2}{(1+\Gamma\omega)}\right\} & -Ra\ a^2 & 0 \\ -1 & \omega+\delta^2+H & -H \\ 0 & -\gamma H & \alpha\omega+\delta^2+\gamma H \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{17}$$

where $\delta^2 = \pi^2 + a^2$ is the total wave number.

For the nontrivial solution of the above matrix equation (17), we require

$$Ra = \frac{\delta^2}{a^2}\left(1+\frac{\omega}{\Pr}+\frac{Da\,\delta^2}{1+\Gamma\omega}\right)\left(\left(\omega+\delta^2\right)+\frac{\left(\alpha\omega+\delta^2\right)H}{\alpha\omega+\delta^2+\gamma H}\right). \tag{18}$$

Setting $\omega = i\,\omega_i$ in Equation (18) and clearing the complex quantities from the denominator, we obtain

$$Ra = \Delta_1 + i\,\omega_i\,\Delta_2, \tag{19}$$

where

$$\Delta_1 = \frac{\delta^2}{a^2}\left(1+\frac{Da\,\delta^2}{1+\Gamma^2\omega_i^2}\right)\left[\delta^2+\frac{\left\{\alpha^2\omega_i^2+\delta^2\left(\delta^2+\gamma H\right)\right\}H}{\alpha^2\omega_i^2+\left(\delta^2+\gamma H\right)^2}\right]$$
$$-\omega_i^2\left[\frac{1}{\Pr}-\frac{Da\delta^2\Gamma}{1+\Gamma^2\omega_i^2}\right]\left[1+\frac{\alpha\gamma H^2}{\alpha^2\omega_i^2+\left(\delta^2+\gamma H\right)^2}\right], \tag{20}$$

$$\Delta_2 = \frac{\delta^2}{a^2}\left[\left(1+\frac{Da\,\delta^2}{1+\Gamma^2\omega_i^2}\right)\left\{1+\frac{\alpha\,\gamma\,H^2}{\alpha^2\omega_i^2+\left(\delta^2+\gamma\,H\right)^2}\right\}\right]$$
$$+\left[\frac{1}{\Pr}-\frac{Da\delta^2\Gamma}{1+\Gamma^2\omega_i^2}\right]\left[\delta^2+\frac{\left\{\alpha^2\omega_i^2+\delta^2\left(\delta^2+\gamma\,H\right)\right\}H}{\alpha^2\omega_i^2+\delta\left(\delta^2+\gamma\,H\right)^2}\right]. \tag{21}$$

Since Ra is a physical quantity, it must be real. Hence, from Equation (21) it follows that either $\omega_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\omega_i \neq 0$, oscillatory onset).

(i) Stationary convection

The steady convection occurs when $\omega_i = 0$ and in that case Eq.(8.30) gives

$$Ra^{St} = \frac{\delta^2}{a^2}(1+Da\delta^2)\left[\frac{\delta^2+H\left(1+\gamma\right)}{\delta^2+\gamma H}\right]. \tag{22}$$

We observe that the expression for Ra^{St} is independent of viscoelastic parameter. Thus, as far as the steady onset is concerned, there is no distinction between the viscous fluid and viscoelastic fluid. We discuss below the results about the oscillatory motions.

(ii) Oscillatroy convection

For oscillatory onset $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives a dispersion relation of the form

$$a_1\left(\omega_i^2\right)^2 + a_2\left(\omega_i^2\right) + a_3 = 0, \quad (23)$$

where

$$a_1 = \Gamma^2\delta^2\alpha^2 + \Gamma^2 H\ \alpha^2 + \Pr\Gamma^2\alpha^2 ,$$

$$a_2 = \delta^2\alpha^2 + H\alpha^2 + \Gamma^2\delta^2(\delta^2+\gamma H)^2 + \Gamma^2 H\delta^2(\delta^2+\gamma H) - \Pr Da\Gamma\delta^4\alpha^2 - \Pr Da\delta^2 H\alpha^2\Gamma + \Pr Da\delta^2\alpha^2 + \Pr\alpha^2 + \Pr\Gamma^2(\delta^2+\gamma H)^2 + \Pr\Gamma^2\alpha\gamma H^2,$$

$$a_3 = \delta^2(\delta^2+\gamma H)^2 + H\delta^2(\delta^2+\gamma H) - \Pr Da\delta^4(\delta^2+\gamma H)^2\Gamma - \Pr Da\delta^4 H\Gamma(\delta^2+\gamma H) + \Pr Da\delta^2(\delta^2+\gamma H)^2 + \Pr Da\delta^2\alpha\gamma H^2 + \Pr(\delta^2+\gamma H)^2 + \Pr\alpha\gamma H^2.$$

Now Equation (23) with $\Delta_2 = 0$, gives

$$Ra_{osc} = \frac{\delta^2}{a^2}\left(1+\frac{Da\,\delta^2}{1+\Gamma^2\omega_i^2}\right)\left[\delta^2 + \frac{\left\{\alpha^2\omega_i^2+\delta^2\left(\delta^2+\gamma H\right)\right\}H}{\alpha^2\omega_i^2+\left(\delta^2+\gamma H\right)^2}\right] + \omega_i^2\left[\frac{1}{\Pr} - \frac{Da\delta^2\Gamma}{1+\Gamma^2\omega_i^2}\right]\left[1+\frac{\alpha\gamma H^2}{\alpha^2\omega_i^2+\left(\delta^2+\gamma H\right)^2}\right], \quad (24)$$

[Conclusions

Thermal convection in a horizontal sparsely packed porous layer saturated with a viscoelastic fluid of the Maxwell type when the fluid and solid phases are not in local thermal equilibrium is investigated analytically. The Lapwood-Brinkman model is used for the momentum equation. The conditions for the onset of stationary and oscillatory convection are derived using a linear stability theory. Linear stability analysis suggests that, there is a competition between the processes of viscous relaxation and thermal diffusion that causes the first convective instability to be oscillatory rather than stationary. It is found that the effect of increasing conductivity ratio γ is to decrease the critical Rayleigh number. The critical wavenumber for the oscillatory mode decreases with increasing value of γ for moderate values of *H,* but it increases with increasing value of γ for large *H*. The critical Rayleigh number is independent of the interphase heat transfer coefficient *H* for the value of $\gamma \geq 10$. The critical Rayleigh number is independent of γ

for small *H,* while for intermediate values of H, it decreases with increasing γ. The increase in the thermal diffusivity ratio α is to increase the critical Rayleigh number for the overstable mode and thus their effect is to delay the onset of convection. For a fixed value of H, the effect of increasing stress relaxation time Γ is to decrease the critical Rayleigh number. The critical wave number remains constant for small and large values of H and for the intermediate values it attains the maximum value. The effect of increasing Prandtl number is to destabilize the system.

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