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Direct product of Intuitionistic fuzzy BG-Ideals in BG-Algebra

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Abstract:

In this paper, we introduced the concept of direct product of intuitionistic fuzzy BG-ideal in BG-algebra and investigate some of their basic properties.

Keywords:

BG-algebra, fuzzy BG-ideal, intuitionistic fuzzy BG-ideal, direct product of intuitionistic fuzzy BG-ideal.

1.Introduction:

In 1965, Zadeh[9] introduced the notion of a fuzzy subset of a set as a method of representing uncertainty in real physical world. The concept of intuitionistic fuzzy subset was introduced by Atanassov[3] in 1986, which is a generalization of the notion of fuzzy sets. In 1966, Imai and Iseki[6] introduced the two classes of abstract algebras, viz., BCK/BCI-algebra. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebras. Later the author have studied direct product of doubt intuitionistic fuzzy ideals in BCI/BCK-algebra. Neggers and Kim[8] introduced a new concept, called B-algebras, which are related to several classes of algebra such as BG-algebra. Zarandi and Saeid[10] developed intuitionistic fuzzy ideal of BG-algebra. In 2019, R.Angelin Suba and K.R.Sobha[1] introduced the new concept of absolute direct product of doubt intuitionistic fuzzy K-ideals in BCK/BCI-algebra. In this paper, we investigate some properties of direct product of intuitionistic fuzzy BG-ideal in BG-algebra.

2.preliminaries

Definition:2.1

A BG-algebra is a non empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(i) $x * x = 0$

(ii) $x * 0 = x$

(iii) $(x * y) * (0 * y) = x \ \forall x, y \in X$.

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For brevity we also call X BG-algebra. A binary relation '$\leq$' on X can be defined by $x \leq y$ if and only if $x * y = 0$.

A non-empty set S of a BG-algebra X is called a subalgebra of X if $x * y \epsilon S$ $\forall$ $x, y \epsilon S$.

Definition:2.2

A fuzzy set μ in X is called a fuzzy BG-ideal of X if it satisfies the following condition:

(i) $\mu(0) \geq \mu(x)$

$(ii) \mu(x) \geq min\{\mu(x * y), \mu(y)\}$ $\forall x, y \epsilon X$.

Definition:2.3

If $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ is an intuitionistic fuzzy sets of BG-Algebra $X \times Y$ is said to be a intuitionistic fuzzy BG-ideal of $X \times Y$ if it satisfies the following axioms

(i) $\mu_{A\times B}(0,0) \geq \mu_{A\times B}(x, y)$

(ii) $\mu_{A\times B}(x_1, y_1) \geq \min\{\mu_{A\times B}((x_1, y_1) * (x_2, y_2)), \mu_{A\times B}(x_2, y_2)\}$

(iii) $\mu_{A\times B}((x_1, y_1) * (x_2, y_2)) \geq \min\{\mu_{A\times B}((x_1, y_1)), \mu_{A\times B}(x_2, y_2)\}$

(i) $\gamma_{A\times B}(0,0) \leq \gamma_{A\times B}(x, y)$

(ii) $\gamma_{A\times B}(x_1, y_1) \leq \max\{\gamma_{A\times B}((x_1, y_1) * (x_2, y_2)), \gamma_{A\times B}(x_2, y_2)\}$

(iii) $\gamma_{A\times B}((x_1, y_1) * (x_2, y_2)) \leq \max\{\gamma_{A\times B}((x_1, y_1)), \gamma_{A\times B}(x_2, y_2)\}$

$\forall$ x_1, x_2, y_1, y_2 ϵX.

3. Direct product of Intuitionistic fuzzy BG-Ideal

Definition:3.1

Let X and Y be BG-algebra and let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X and Y respectively.Then the direct product of intuitionistic fuzzy sets A and B is defined by $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ where $\mu_{A\times B}: X \times Y \to [0,1]$ is given by
$\mu_{A\times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ and $\gamma_{A\times B}: X \times Y \to [0,1]$ is given by
$\gamma_{A\times B}(x, y) = \max\{\gamma_A(x), \gamma_B(y)\}$ for all $(x, y) \epsilon X \times Y$.

Theorem:3.2

If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively. Then $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ is a intuitionistic fuzzy BG-ideal in BG-algebra
X and Y.
Soln:

For any $(x, y) \epsilon$ $X \times Y$

$\mu_{A\times B}(0,0) = \min\{\mu_{A\times B}(0), \mu_{A\times B}(0)\}$

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$$\geq \min\{\mu_{A\times B}(x), \mu_{A\times B}(y)\}$$

$$\geq \mu_{A\times B}(x,y)$$

$$\gamma_{A\times B}(0,0) = \max\{\gamma_{A\times B}(0), \gamma_{A\times B}(0)\}$$

$$\leq \max\{\gamma_{A\times B}(x), \gamma_{A\times B}(y)\}$$

$$\leq \gamma_{A\times B}(x,y)$$

Now for any $(x_1,y_1)\,,(x_2,y_2)\epsilon X\times Y$

$$\mu_{A\times B}(x_1,y_1) \geq \min\{\mu_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big), \mu_{A\times B}\,(x_2,y_2)\}$$

And

$$\gamma_{A\times B}(x_1,y_1) \leq \max\{\gamma_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big), \gamma_{A\times B}\,(x_2,y_2)\}$$

$$\mu_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big) = \min\{\mu_{A\times B}\Big(\big((x_1,y_1)*(x_2,y_2)\big)*(x_2,y_2)\Big), \mu_{A\times B}\,(x_2,y_2)\}$$

$$= \min\{\mu_{A\times B}\Big(\big((x_1,y_1)*(x_2,y_2)\big)*\big((0,0)*(x_2,y_2)\big)\Big), \mu_{A\times B}\,(x_2,y_2)\}$$

$$\mu_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big) \geq \min\{\mu_{A\times B}\big((x_1,y_1)\big), \mu_{A\times B}\,(x_2,y_2)\}$$

And

$$\gamma_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big) = \max\{\gamma_{A\times B}\Big(\big((x_1,y_1)*(x_2,y_2)\big)*(x_2,y_2)\Big), \gamma_{A\times B}\,(x_2,y_2)\}$$

$$= \max\{\gamma_{A\times B}\Big(\big((x_1,y_1)*(x_2,y_2)\big)*\big((0,0)*(x_2,y_2)\big)\Big), \gamma_{A\times B}\,(x_2,y_2)\}$$

$$\gamma_{A\times B}\big((x_1,y_1)*(x_2,y_2)\big) \leq \max\{\gamma_{A\times B}\big((x_1,y_1)\big), \gamma_{A\times B}\,(x_2,y_2)\}$$

Hence $A\times B = (\mu_{A\times B}, \gamma_{A\times B})$ is a intuitionistic fuzzy BG-ideal in BG-algebra $X\times Y$.

Example :3.3

Let $A = (\mu_A, \gamma_A)$ be a intuitionistic fuzzy BG-ideal of X as defined by

X	0	1	2	3	4
μ_A	0.76	0.65	0.42	0.42	0.65
γ_A	0.22	0.33	0.56	0.56	0.33

Let $B = (\mu_B, \gamma_B)$ be a intuitionistic fuzzy BG-ideal of X as defined by

X	0	1	2	3	4
μ_B	0.77	0.50	0.56	0.50	0.50
γ_B	0.21	0.48	0.42	0.48	0.48

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Clearly $X \times X$ is also a BG-Algebra

Here

$\mu_{A\times B}(0,0) = 0.76,$

$\mu_{A\times B}(1,0) = \mu_{A\times B}(4,0) = 0.65$

$\mu_{A\times B}(2,0) = \mu_{A\times B}(3,0) = \mu_{A\times B}(2,1) = \mu_{A\times B}(2,3) = \mu_{A\times B}(2,4) =$

$\mu_{A\times B}(2,2) = \mu_{A\times B}(3,1) = \mu_{A\times B}(3,3) = \mu_{A\times B}(3,4) = \mu_{A\times B}(3,2) = 0.42$

$\mu_{A\times B}(0,2) = \mu_{A\times B}(1,2) = \mu_{A\times B}(4,2) = 0.56$

$\mu_{A\times B}(0,1) = \mu_{A\times B}(0,3) = \mu_{A\times B}(0,4) = \mu_{A\times B}(4,1) = \mu_{A\times B}(4,3) =$

$\mu_{A\times B}(4,4) = \mu_{A\times B}(1,1) = \mu_{A\times B}(1,3) = \mu_{A\times B}(1,4) = 0.50$

Also, $\gamma_{A\times B}(0,0)$ =0.22

$\gamma_{A\times B}(0,1) = \gamma_{A\times B}(0,3) = \gamma_{A\times B}(1,1) = \gamma_{A\times B}(1,3) = \gamma_{A\times B}(1,4) =$

$\gamma_{A\times B}(4,1) = \gamma_{A\times B}(4,3) = \gamma_{A\times B}(4,4) = 0.48$

$\gamma_{A\times B}(0,2) = \gamma_{A\times B}(1,2) = \gamma_{A\times B}(4,2) = \gamma_{A\times B}(0,4) = 0.50$

$\gamma_{A\times B}(4,0) = \gamma_{A\times B}(1,0) = 0.33$

$\gamma_{A\times B}(2,1) = \gamma_{A\times B}(2,2) = \gamma_{A\times B}(2,3) = \gamma_{A\times B}(2,0) = \gamma_{A\times B}(2,4) =$

$\gamma_{A\times B}(3,,0) = \gamma_{A\times B}(3,1) = \gamma_{A\times B}(3,2) = \gamma_{A\times B}(3,3) = \gamma_{A\times B}(3,4) = 0.56$

Therefore $A \times B$ is a intuitionistic fuzzy BG-ideal of $X \times X$.

Lemma:3.4

Let $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ be a intuitionistic fuzzy BG-ideal in BG-algebra $X \times Y$ .If $(x, y) \leq (a, b)$ then $\mu_{A\times B}(x, y) \geq \mu_{A\times B}(a, b)$ and $\gamma_{A\times B}(x, y) \leq \gamma_{A\times B}(a, b)$ for every $(a, b), (x, y) \in X \times Y$.

Soln:

Let $(a, b), (x, y) \in X \times Y$

And $(x, y) \leq (a, b)$ implies $(x, y) * (a, b) = (0,0)$

$\mu_{A\times B}(x, y) = \mu_{A\times B}((x, y) * (0,0))$

$\geq \min\{\mu_{A\times B}(x, y) * (0,0) * (a, b),\ \mu_{A\times B}(a, b)\}$

$\geq \min\{\mu_{A\times B}(x, y) * (a, b),\ \mu_{A\times B}(a, b)\}$

$\geq \min\{\mu_{A\times B}(0,0),\ \mu_{A\times B}(a, b)\}$

$\geq \mu_{A\times B}(a, b)$

Similarly, $\gamma_{A\times B}(x, y) \leq \gamma_{A\times B}(a, b)$

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Theorem:3.5

If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively.Then $\Pi(A \times B) = (\mu_{A\times B}, \bar{\mu}_{A\times B})$ is a intuitionistic fuzzy BG-ideal of $X \times Y$, where $\bar{\mu}_{A\times B} = 1 - \mu_{A\times B}$.

Proof :

By thrm:3.2, $A \times B$ is a intuitionistic fuzzy BG-ideal of $X \times Y$.

Therefore for any $(a, b) \epsilon X \times Y$,

$\mu_{A\times B}(0,0) \geq \mu_{A\times B}(a, b)$

That is $1 - \mu_{A\times B}(0,0) \leq 1 - \mu_{A\times B}(a, b)$

That is $\bar{\mu}_{A\times B}(0,0) \leq \bar{\mu}_{A\times B}(a, b)$

Now for any $(x_1, y_1), (x_2, y_2) \epsilon X \times Y$

$\mu_{A\times B}(x_1, y_1) \geq min\{\mu_{A\times B}(x_1, y_1) * (x_2, y_2), \mu_{A\times B}(x_2, y_2)\}$

$1 - \mu_{A\times B}(x_1, y_1) \leq 1 - min\{\mu_{A\times B}(x_1, y_1) * (x_2, y_2), \mu_{A\times B}(x_2, y_2)\}$

$\bar{\mu}_{A\times B}(x_1, y_1) \leq max\{\bar{\mu}_{A\times B}(x_1, y_1) * (x_2, y_2), \bar{\mu}_{A\times B}(x_2, y_2)\}$

And

$\mu_{A\times B}((x_1, y_1) * (x_2, y_2)) \geq min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\}$

$1 - \mu_{A\times B}\big((x_1, y_1) * (x_2, y_2)\big) \leq 1 - min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\}$

$\bar{\mu}_{A\times B}\big((x_1, y_1) * (x_2, y_2)\big) \leq max\{\bar{\mu}_{A\times B}(x_1, y_1), \bar{\mu}_{A\times B}(x_2, y_2)\}$

Hence $\Pi(A \times B) = (\mu_{A\times B}, \bar{\mu}_{A\times B})$ is a intuitionistic fuzzy BG-ideal of $X \times Y$.

Theorem3.6

If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively.Then $\Pi(A \times B) = (\gamma_{A\times B}, \bar{\gamma}_{A\times B})$ is a intuitionistic fuzzy BG-ideal of $X \times Y$, where $\bar{\gamma}_{A\times B} = 1 - \gamma_{A\times B}$.

Proof:

By thrm:3.2, $A \times B$ is a intuitionistic fuzzy BG-ideal of $X \times Y$.

Therefore for any $(a, b) \epsilon X \times Y$,

$\gamma_{A\times B}(0,0) \leq \gamma_{A\times B}(a, b)$

That is $1 - \gamma_{A\times B}(0,0) \geq 1 - \gamma_{A\times B}(a, b)$

That is $\bar{\gamma}_{A\times B}(0,0) \geq \bar{\gamma}_{A\times B}(a, b)$

Now for any $(x_1, y_1), (x_2, y_2) \epsilon X \times Y$

$\gamma_{A\times B}(x_1, y_1) \leq max\{\gamma_{A\times B}(x_1, y_1) * (x_2, y_2), \gamma_{A\times B}(x_2, y_2)\}$

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$1 - \gamma_{A\times B}(x_1, y_1) \geq 1 - max\{\gamma_{A\times B}(x_1, y_1) * (x_2, y_2), \gamma_{A\times B}(x_2, y_2)\}$

$\bar{\gamma}_{A\times B}(x_1, y_1) \geq min\{\bar{\gamma}_{A\times B}(x_1, y_1) * (x_2, y_2), \bar{\gamma}_{A\times B}(x_2, y_2)\}$

And

$\gamma_{A\times B}((x_1, y_1) * (x_2, y_2)) \leq max\{\gamma_{A\times B}(x_1, y_1), \gamma_{A\times B}(x_2, y_2)\}$

$1 - \gamma_{A\times B}\big((x_1, y_1) * (x_2, y_2)\big) \geq 1 - max\{\gamma_{A\times B}(x_1, y_1), \gamma_{A\times B}(x_2, y_2)\}$

$\bar{\gamma}_{A\times B}\big((x_1, y_1) * (x_2, y_2)\big) \geq min\{\bar{\gamma}_{A\times B}(x_1, y_1), \bar{\gamma}_{A\times B}(x_2, y_2)\}$

Hence $\Pi(A \times B)$ is a intuitionistic fuzzy BG-ideal of $\times Y$.

Lemma:3.7

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively. If $(A \times B)$ is a intuitionistic fuzzy BG-ideal of $X \times Y$, then the following are true

(i) $\mu_A(0) \geq \mu_B(y)$ and

$\mu_B(0) \geq \mu_A(x)\ \forall\ x\epsilon X, y\epsilon Y.$

(ii) $\gamma_A(0) \leq \gamma_B(y)$ and

$\gamma_B(0) \leq \gamma_A(x)\ \forall\ x\epsilon X, y\epsilon Y.$

Proof:

Assume $\mu_A(0) < \mu_B(y)$ and $\mu_B(0) < \mu_A(x)$ for some $x\epsilon X, y\epsilon Y$.

Then $\mu_{A\times B}(x, y) = min\{\mu_A(x), \mu_B(y)\}$

$\geq min\{\mu_B(0), \mu_A(0)\}$

$\geq min\{\mu_A(0), \mu_B(0)\}$

$= \mu_{A\times B}(0,0)$

This is a contradiction

Similarly, Let $\gamma_A(0) > \gamma_B(y)$ and $\gamma_B(0) > \gamma_A(x)$ for some $x\epsilon X, y\epsilon Y$.

Then $\gamma_{A\times B}(x, y) = max\{\gamma_A(x), \gamma_B(y)\}$

$\leq max\{\gamma_B(0), \gamma_A(0)\}$

$\leq max\{\gamma_A(0), \gamma_B(0)\}$

$= \gamma_{A\times B}(0,0)$

This is a contradiction.

Hence the result is proved.

Definition:3.8

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Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively. Then the intersection $A \cap B$ is defined as $A \cap B = (\mu_{A\cap B}, \gamma_{A\cap B})$ where $\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(y): x\epsilon X\}$ and $\gamma_{A\cap B}(x) = max\{\gamma_A(x), \gamma_B(y): x\epsilon X\}$.

Theorem:3.9

Let $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ and $C \times D = (\mu_{C\times D}, \gamma_{C\times D})$ is a intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively.Then $(A \times B) \cap (c \times D) = (\mu_{(A\times B)\cap(c\times D)}, \gamma_{(A\times B)\cap(c\times D)})$ is a intuitionistic fuzzy BG-ideal of BG-algebra $X \times Y$.

Proof:

For any $(x, y)\epsilon X \times Y$, it gives that

(i) $\mu_{A\times B}(0,0) \geq \mu_{A\times B}(x, y)$ and $\mu_{C\times D}(0,0) \geq \mu_{C\times D}(x, y)$

$\{ \mu_{A\times B}(0,0), \mu_{C\times D}(0,0)\} \geq \{\mu_{A\times B}(x, y), \mu_{C\times D}(x, y)\}$

$min\{ \mu_{A\times B}(0,0), \mu_{C\times D}(0,0)\} \geq min\{\mu_{A\times B}(x, y), \mu_{C\times D}(x, y)\}$

$\mu_{(A\times B)\cap(c\times D),}(0,0) \geq \mu_{(A\times B)\cap(c\times D),}(x, y)$.

(ii) $\gamma_{A\times B}(0,0) \leq \gamma_{A\times B}(x, y)$ and $\gamma_{C\times D}(0,0) \leq \gamma_{C\times D}(x, y)$

$\{ \gamma_{A\times B}(0,0), \gamma_{C\times D}(0,0)\} \leq \{\gamma_{A\times B}(x, y), \gamma_{C\times D}(x, y)\}$

$max\{ \gamma_{A\times B}(0,0), \gamma_{C\times D}(0,0)\} \leq max\{\gamma_{A\times B}(x, y), \gamma_{C\times D}(x, y)\}$

$\gamma_{(A\times B)\cap(c\times D),}(0,0) \leq \gamma_{(A\times B)\cap(c\times D),}(x, y)$.

(iii)For all $(x_1, y_1), (x_2, y_2)\epsilon X \times Y$

$\mu_{A\times B}\{(x_1, y_1), (x_2, y_2)\} \geq min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\}$

$\mu_{C\times D}\{(x_1, y_1), (x_2, y_2)\} \geq min\{\mu_{C\times D}(x_1, y_1), \mu_{C\times D}(x_2, y_2)\}$

$$\{\mu_{A\times B}\{(x_1, y_1), (x_2, y_2)\}, \mu_{C\times D}\{(x_1, y_1), (x_2, y_2)\}\} \geq \begin{Bmatrix} min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\} \\ min\{\mu_{C\times D}(x_1, y_1), \mu_{C\times D}(x_2, y_2)\} \end{Bmatrix}$$

$$min\{\mu_{A\times B}\{(x_1, y_1), (x_2, y_2)\}, \mu_{C\times D}\{(x_1, y_1), (x_2, y_2)\}\} \geq min \begin{Bmatrix} min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\} \\ min\{\mu_{C\times D}(x_1, y_1), \mu_{C\times D}(x_2, y_2)\} \end{Bmatrix}$$

$$min\{\mu_{A\times B}\{(x_1, y_1), (x_2, y_2)\}, \mu_{C\times D}\{(x_1, y_1), (x_2, y_2)\}\} \geq min \begin{Bmatrix} min\{\mu_{A\times B}(x_1, y_1), \mu_{C\times D}(x_1, y_1)\} \\ min\{\mu_{A\times B}(x_2, y_2), \mu_{C\times D}(x_2, y_2)\} \end{Bmatrix}$$

$\mu_{(A\times B)\cap(c\times D)}(x_1, y_1) * (x_2, y_2) \geq min\{\mu_{(A\times B)\cap(c\times D)}(x_1, y_1), \mu_{(A\times B)\cap(c\times D)}(x_2, y_2)\}$.

Similarly,

(iv) $\gamma_{(A\times B)\cap(c\times D)}(x_1, y_1) * (x_2, y_2) \leq max\{\gamma_{(A\times B)\cap(c\times D)}(x_1, y_1), \gamma_{(A\times B)\cap(c\times D)}(x_2, y_2)\}$

(v) $\mu_{A\times B}(x_1, y_1) \geq min\{\mu_{A\times B}((x_1, y_1) * (x_2, y_2)), \mu_{A\times B}(x_2, y_2)\}$

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$$\mu_{c\times D}(x_1,y_1) = min\{\mu_{C\times D}((x_1,y_1) * (x_2,y_2)), \mu_{c\times D}(x_2,y_2)\}$$

$$\{\mu_{A\times B}(x_1,y_1), \mu_{c\times D}(x_1,y_1)\} = \begin{Bmatrix} min\{\mu_{A\times B}(x_1,y_1) * (x_2,y_2), \mu_{A\times B}(x_2,y_2)\}, \\ min\{\mu_{C\times D}(x_1,y_1) * (x_2,y_2), \mu_{c\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$\{\mu_{A\times B}(x_1,y_1), \mu_{c\times D}(x_1,y_1)\} = \begin{Bmatrix} min\{\mu_{A\times B}((x_1,y_1) * (x_2,y_2)), \mu_{C\times D}((x_1,y_1) * (x_2,y_2))\}, \\ min\{\mu_{A\times B}(x_2,y_2), \mu_{c\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$\{\mu_{(A\times B)\cap(c\times D)}(x_1,y_1)\} \geq \begin{Bmatrix} min\{\mu_{(A\times B)\cap(c\times D)}((x_1,y_1) * (x_2,y_2))\}, \\ min\{\mu_{(A\times B)\cap(c\times D)}(x_2,y_2)\} \end{Bmatrix}$$

Similarly (vi)

$$\{\gamma_{(A\times B)\cap(c\times D)}(x_1,y_1)\} \leq \begin{Bmatrix} max\{\gamma_{(A\times B)\cap(c\times D)}((x_1,y_1) * (x_2,y_2))\}, \\ max\{\gamma_{(A\times B)\cap(c\times D)}(x_2,y_2)\} \end{Bmatrix}$$

Hence $(A \times B) \cap (c \times D) = (\mu_{(A\times B)\cap(c\times D),}\gamma_{(A\times B)\cap(c\times D)})$ is a intuitionistic fuzzy BG-ideal of BG-algebra $X \times Y$.

Definition:3.10

Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively.Then the union $A \cup B$ is defined as $A \cup B = (\mu_{A\cup B}, \gamma_{A\cup B})$ where $\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(y): x\epsilon X\}$ and $\gamma_{A\cup B}(x) = min\{\gamma_A(x), \gamma_B(y): x\epsilon X\}$

Theorem:3.11

Let $A \times B = (\mu_{A\times B}, \gamma_{A\times B})$ and $C \times D = (\mu_{C\times D}, \gamma_{C\times D})$ is a intuitionistic fuzzy BG-ideal in BG-algebra X and Y respectively.Then $(A \times B) \cup (c \times D) = (\mu_{(A\times B)\cup(c\times D),}\gamma_{(A\times B)\cup(c\times D)})$ is a intuitionistic fuzzy BG-ideal of BG-algebra $X \times Y$.

Proof:

For any $(x,y)\epsilon X \times Y$, it gives that

(i) $\mu_{A\times B}(0,0) \geq \mu_{A\times B}(x,y)$ and $\mu_{C\times D}(0,0) \geq \mu_{C\times D}(x,y)$

$\{ \mu_{A\times B}(0,0), \mu_{C\times D}(0,0)\} \geq \{\mu_{A\times B}(x,y), \mu_{C\times D}(x,y)\}$

$min\{ \mu_{A\times B}(0,0), \mu_{C\times D}(0,0)\} \geq min\{\mu_{A\times B}(x,y), \mu_{C\times D}(x,y)\}$

$\mu_{(A\times B)\cup(c\times D),}(0,0) \geq \mu_{(A\times B)\cup(c\times D),}(x,y)$.

(ii) $\gamma_{A\times B}(0,0) \leq \gamma_{A\times B}(x,y)$ and $\gamma_{C\times D}(0,0) \leq \gamma_{C\times D}(x,y)$

$\{ \gamma_{A\times B}(0,0), \gamma_{C\times D}(0,0)\} \leq \{\gamma_{A\times B}(x,y), \gamma_{C\times D}(x,y)\}$

$max\{ \gamma_{A\times B}(0,0), \gamma_{C\times D}(0,0)\} \leq max\{\gamma_{A\times B}(x,y), \gamma_{C\times D}(x,y)\}$

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$\gamma_{(A\times B)\cup(c\times D),}(0,0) \leq \gamma_{(A\times B)\cup(c\times D),}(x,y)$.

(iii)For all $(x_1,y_1),(x_2,y_2)\epsilon X\times Y$

$\mu_{A\times B}\{(x_1,y_1),(x_2,y_2)\} \geq min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\}$

$\mu_{C\times D}\{(x_1,y_1),(x_2,y_2)\} \geq min\{\mu_{C\times D}(x_1,y_1),\mu_{C\times D}(x_2,y_2)\}$

$$\{\mu_{A\times B}\{(x_1,y_1),(x_2,y_2)\},\mu_{C\times D}\{(x_1,y_1),(x_2,y_2)\}\} \geq \begin{Bmatrix} min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\} \\ min\{\mu_{C\times D}(x_1,y_1),\mu_{C\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$min\{\mu_{A\times B}\{(x_1,y_1),(x_2,y_2)\},\mu_{C\times D}\{(x_1,y_1),(x_2,y_2)\}\} \geq min\begin{Bmatrix} min\{\mu_{A\times B}(x_1,y_1),\mu_{A\times B}(x_2,y_2)\} \\ min\{\mu_{C\times D}(x_1,y_1),\mu_{C\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$min\{\mu_{A\times B}\{(x_1,y_1),(x_2,y_2)\},\mu_{C\times D}\{(x_1,y_1),(x_2,y_2)\}\} \geq min\begin{Bmatrix} min\{\mu_{A\times B}(x_1,y_1),\mu_{C\times D}(x_1,y_1)\} \\ min\{\mu_{A\times B}(x_2,y_2),\mu_{C\times D}(x_2,y_2)\} \end{Bmatrix}$$

$\mu_{(A\times B)\cup(c\times D)}(x_1,y_1)*(x_2,y_2) \geq min\{\mu_{(A\times B)\cup(c\times D)}(x_1,y_1),\mu_{(A\times B)\cup(c\times D)}(x_2,y_2)\}$.

Similarly,

(iv) $\gamma_{(A\times B)\cup(c\times D)}(x_1,y_1)*(x_2,y_2) \leq max\{\gamma_{(A\times B)\cup(c\times D)}(x_1,y_1),\gamma_{(A\times B)\cup(c\times D)}(x_2,y_2)\}$

(v) $\mu_{A\times B}(x_1,y_1) \geq min\{\mu_{A\times B}((x_1,y_1)*(x_2,y_2)),\mu_{A\times B}(x_2,y_2)\}$

$$\mu_{c\times D}(x_1,y_1) = min\{\mu_{c\times D}((x_1,y_1)*(x_2,y_2)),\mu_{c\times D}(x_2,y_2)\}$$

$$\{\mu_{A\times B}(x_1,y_1),\mu_{c\times D}(x_1,y_1)\} = \begin{Bmatrix} min\{\mu_{A\times B}(x_1,y_1)*(x_2,y_2),\mu_{A\times B}(x_2,y_2)\}, \\ min\{\mu_{C\times D}(x_1,y_1)*(x_2,y_2),\mu_{c\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$\{\mu_{A\times B}(x_1,y_1),\mu_{c\times D}(x_1,y_1)\} = \begin{Bmatrix} min\{\mu_{A\times B}((x_1,y_1)*(x_2,y_2)),\mu_{C\times D}((x_1,y_1)*(x_2,y_2))\}, \\ min\{\mu_{A\times B}(x_2,y_2),\mu_{c\times D}(x_2,y_2)\} \end{Bmatrix}$$

$$\{\mu_{(A\times B)\cup(c\times D)}(x_1,y_1)\} \geq \begin{Bmatrix} min\{\mu_{(A\times B)\cup(c\times D)}((x_1,y_1)*(x_2,y_2))\}, \\ min\{\mu_{(A\times B)\cup(c\times D)}(x_2,y_2)\} \end{Bmatrix}$$

Similarly (vi)

$$\{\gamma_{(A\times B)\cup(c\times D)}(x_1,y_1)\} \leq \begin{Bmatrix} max\{\gamma_{(A\times B)\cup(c\times D)}((x_1,y_1)*(x_2,y_2))\}, \\ max\{\gamma_{(A\times B)\cup(c\times D)}(x_2,y_2)\} \end{Bmatrix}$$

Hence $(A\times B)\cup(c\times D) = (\mu_{(A\times B)\cup(c\times D),}\gamma_{(A\times B)\cup(c\times D)})$ is a intuitionistic fuzzy BG-ideal of BG-algebra $X\times Y$.

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Reference:

[1]. R.Angelin suba and ,K.R.Sobha,Absolute direct product of doubt intuitionistic fuzzy K-ideals in BCK/BCI-algebras,International journal of applied engineering research,ISSN 0973-4562 volume 14,Number 8, 2019.

[2]. R.Angelin suba and ,K.R.Sobha, upper and lower level sets of absolute direct product of doubt intuitionistic fuzzy K-Ideal of BCK/BCI-algebra, International Journal of Research in engineering and Science volume 9 Issue 3/2021/pp.12-15.

[3]. Atanassov, K.T, (1986); Intuitionistic fuzzy sets, Fuzzy sets and systems, 20 :87-96

[4]. T. Bej and M.Pal, Direct product of Doubt intuitionistic fuzzy ideals in BCK/BCI –algebras, International Journal of Pure and applied Researches; 2016 vol. 2 (1), ISSN: 2455-474X

[5]. T. Bej and M.Pal.,(2015) Doubt intuitionistic fuzzy ideals in BCK/BCI –algebras, International Journal of Fuzzy logic systems, 5(1) : 1-13

[6]. Y.Imai, K.Iseki, On axiom system of propositional calculi, XIV, proceeding of the Japan academy,1966.pp. 19-22.

[7]. C.B.Kim and H.S.Kim, On BG-algebras, Demonstraction Mathematica 41 (2008) 497-505.

[8]. J.Neggers, H.S.Kim, On B-algebra, Math.Vensik 54 (2002) 21-29.

[9]. L.A.zadeh, Fuzzy sets, information and Control 8 (1965) 338-353.

[10]. A.Zarandi, A.B.Saeid, On intuitionistic fuzzy ideals of BG-algebra, World Academy of Sciences Engineering an Technology 5 (2005) 187-189