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SOME BASIC PROPERTIES OF L-IDEALS AND CONGRUENCES IN LATTICE ORDERED COMMUTATIVE LOOPS

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Abstract: This manuscript illustrates the significance of a normal subloop, l-morphism, l-ideal of an l-loop also we have succeeded in determining a corresponding congruence relation on the l-loop and establishing a one-to-one correspondence between the l-ideals and congruence relations of an l-loop A.

Key words: Loops, partial order, lattices, ordered abelian groups, Ideals-congruence relations

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1. INTRODUCTION:

In 1967, G. Birkhoff in Lattice Theory, various properties of lattice ordered groups were established. In 1970, T. Evans described about lattice ordered loops and quasi-groups. In 1990, Hala made a description on quasi-groups and loops [1-3]. In view of this a lot of interest has been shown different authors develop these concepts in different algebraic systems. In 2014, V. B. V. N. Prasad and J. Venkateswara Rao, were gave Categorization of Normal Sub Loop and Ideal of Loops[4].

2. l-Ideals:

Definition 2.1: Let A be an l-loop. For a, b∈A, we define $a*b= (a-b)\vee(b-a) =a\vee b-a\wedge b$.

Lemma 2.1: Let A be a l-loop. Then ∀a, b ∈A

(i) $a*b \geq 0$ with equality if and only if a=b.
(ii) $a*b=b*a$.
(iii) $(a\vee b) *(a\wedge b) =a*b$.

Proof: The proof follows easily.

Definition 2.2: A sub loop R of a loop A is called a normal sub loop of A [2], if and only if

$$(R+x) +y=R+(x+y), \forall x, y \in A.$$

It is not hard to see that a normal sub loop R of a loop A partitions it.

Definition 2.3: A nonempty subset R of an l-loop A is called a l-ideal if and only if R is a normal sub loop of A in which a∈R, b∈A, $b*0\leq a*0 \Rightarrow b\in R$.

Theorem 2.1: A nonempty subset R of a l-loop A is an l-ideal if and only if R is a convex normal l-sub loop of A.

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Proof: Let R be an l-ideal and $a\le x\le b$, $a, b\in R$.

Then $a\wedge b\le x\le a\vee b$.

Now $x*0\le(a\vee b)\vee(0-a\wedge b)$

$=(a\vee b)\vee(0-a)\vee(0-b)=(a*0)\vee(b*0)\le a*0+b*0$.

$\Rightarrow x\in R$.

Hence R is convex.

Also $(a\vee b)*0=(a\vee b)\vee(0-a\vee b)\le a*0\vee b*0$

$\le a*0+b*0$

$\Rightarrow a\vee b\in R$,

and $(a\wedge b)*0=(a\wedge b)\vee[0-(a\wedge b)]\le(a*0)\vee(b*0)\le(a*0)+(b*0)$

$\Rightarrow a\wedge b\in R$.

Thus, R is a convex sublattice of A.

Conversely let R be a convex normal l-subloop of A.

Let $a\in R$, $b*0\le a*0$.

Now $b\le b*0\le a*0$

$\Rightarrow 0-b\ge 0-(a*0)$ so that $0-b\in R$ by conversity in R and hence $b=0-(0-b)\in R$.

This completes the proof.

Definition 2.4: An l-morphism of an l-loop A into a l-loop B is a mapping $f: A\longrightarrow B$ such that $\forall a, b\in A$,

(i) $f(a+b)=f(a)+f(b)$,
(ii) $f(a-b)=f(a)-f(b)$,
(iii) $f(a\vee b)=f(a)\vee f(b)$,
(iv) $f(a\wedge b)=f(a)\wedge f(b)$.

Corollary 2.1: If $f: A\longrightarrow B$ is an l-morphism of an l-loop A into an l-loop B, then $\forall a, b\in A$, $f(a*b)=f(a)*f(b)$.

Definition 2.5: An equivalence relation θ on an l-loop A is a congruence relation if for a, b $\in A$, $a\equiv b(\theta)\Rightarrow a+x\equiv b+x(\theta), a-x\equiv b-x(\theta), x-a\equiv x-b(\theta), a\vee x\equiv b\vee x(\theta)$ and $a\wedge x\equiv b\wedge x(\theta), \forall x\in A$.

Since an l-loop A is equationally definable, we have the following:

Theorem 2.2: Let θ be a congruence relation on an l-loop A. For any $a\theta, b\theta\in A/\theta$, define $a\theta+b\theta=(a+b)\theta$, $a\theta-b\theta=(a-b)\theta$ and $a\theta$ is positive iff $a\theta$ contains a positive element. Then $(A/\theta, +, -, \le)$ is an l-loop.

The following theorem establishes a one-one correspondence between the l-ideals and the congruence relations of an l-loop.

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Theorem 2.3: There is one-to-one correspondence between the l-ideals and congruence relations of an l-loop A.

Proof: Let R be an l-ideal of A. Define

(a) a≡b(θ_R) iff a=b or a, b∈R.
Clearly θ_R is an equivalence relation on A.

Next define a relation θ_R' as follows:

(b) a≡b(θ_R') iff ∃ elements t, t', c, s, s'∈ A such that a=[t∨(c+s)]∧t' and b=[t∨(c+s')]∧t' where s≡s'(θ_R).

We know show that θ_R' is reflexive, symmetric and satisfies the substitution property for +, -,∨,∧.

(i) Since a = [(a∧b) ∨ (0+a)]∧(a∨b), it follows that a≡a (θ_R').
(ii) Clearly θ_R' is symmetric.
(iii) Let a≡b (θ_R'). Then ∃ elements t, t', c, s, s'∈A such that a=[t∨(c+s)]∧t' and b=[t∨(c+s')]∧t' where s≡s'(θ_R).
Now x+a = x+t'∧ [t∨(c+s)]
= (x+t') ∧ [(t+x)∨{x+(c+s)}]
= (x+t') ∧ [(t+x)∨{(x+c) +s_1}] for some s_1∈ R.

Similarly, x+b=(x+t')∧[(t+x)∨{(x+c) +s_1'}] where s_1'∈ R.

Thus x+a≡x+b (θ_R').

In our subsequent paper of ideal theory, we have simplified the proof of this theorem. If S is an l-ideal of A then the a≡b(θ_s) ⟺ a-b∈S gives congruence relation.

Again

a-x = t'∧[t∨(c+s)]-x

=(t'-x)∧[(t-x)∨{(c+s)-x}]

=(t'-x)∧[(t-x)∨{(c-s) +s_2}] for some s_2∈ R.

Similarly, b-x=(t'-x)∧[(t-x)∨{(c-x) +s_2'}] where s_2'∈ R.

Thus a-x≡b-x (θ_R').

Now

x-a = x-[t'∨{t∨(c+s)}]

=[x-(t'∧t)]∧[(x-t')∨{(x-(c+s)}]

=[x-(t'∧t)]∧[(x-t')∨{(x-c) +s_3}] for some s_3∈R

Similarly, x-b=[(x-(t'∧t)]∧[(x-t')∨{(x-c) +s_3'}] where s_3'∈ R

Thus x-a≡x-b (θ_R').

Since A is a distributive lattice it is easy to show that (x∨a)≡x∨b (θ_R') and also

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clearly x∧a≡x∧b (θ_R').

Thus θ_R' satisfies substitution property for +, -, ∨, ∧.

Now let θ_R'' be the transitive extension of θ_R'.

It is easy to see that $\theta_R \leq \theta_R' \leq \theta_R''$

Hence an l-ideal R in A defines a congruence relation θ_R'' in A.

Conversely let θ be a congruence relation on A and R be the set of all x≡(0)θ

Clearly a, b ∈ R ⟹ a+b ∈ R.

Let x∈R, y*0≤x*0.

Now x≡0(θ) ⟹ x*0≡0(θ) ⟹ (y*0)∨(x*0)≡0(θ)

⟹ y*0≡0(θ) ⟹ y∨(0-y) ≡ 0(θ) ⟹ y∧0≡y≡y∨0(θ)

⟹ y∨0≡(y∧0)∨0=0(θ) ⟹ $y \equiv 0(\theta)$ ⟹ y∈R.

Hence R is an l-ideal.

Now let θ_R'' be the congruence relation defined by R.

Then a≡b (θ_R'') ⟹ ∃ a sequence a=z_0, z_1, z_2,.. z_n = b in A such that $z_i \equiv z_{i+1}(\theta_R')$.

Then z_i=[t∨(c+s)]∧t',

Z_{i+1}=[t∨(c+s')]∧t' where s≡s'(θ_R) that is s, s'∈ R=0[θ].

Since θ is a congruence relation and s≡s' (θ), $z_i \equiv z_{i+1}(\theta)$ and by transitivity of θ, it follows that a≡b (θ) showing $\theta_R'' \subseteq \theta$.

Again a ≡ b(θ) ⟹ a $-b \equiv 0(\theta)$ ⟹ a-b ∈ R.

⟹a-b ≡ 0(θ_R) ⟹ a ≡ b(θ_R) ⟹ a≡b (θ_R'') ⟹ $\theta \leq \theta_R''$.

$\theta_R'' = \theta$. This completes the proof

Corollary 2.2: The congruence relation on any l-loop A are the partitions of A into the cosets of its different l-ideals.

Theorem 2.4: The congruence relations on any l-loop A is a complete Algebraic Brouwerian lattice.

Proof: Follows in the same lines as in l-groups[l].

Corollary 2.3: The l-ideals of an l-loop A forms a complete Algebraic Brouwerian lattice.

3. CONCLUSION:

In this paper some important aspects in lattice ordered algebraic structures and especially some more properties of l-ideals and congruence's in lattice ordered loops were established. Further there is so much scope for the remaining algebraic structures in lattice ordered loops.

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