Numerical study of Chemical and Radiation absorption on convective Heat and Mass Transfer through a porous medium In a Channel

Dhananjaiah D.S

Dept of Mathematics, Government first Grade College, K.R.Pet, Mandya, India

ABSTRACT

 We analyse the effect of chemical reaction and radiation absorption on unsteady convective Heat and Mass Transfer flow of viscous fluid through a Porous Medium in a vertical channel in the presence of Heat generating sources. The unsteadiness in the flow is due to a traveling thermal wave imposed on the boundary y=+L. Taking the aspect ratio δ <<1 the non linear coupled governing equations are solved by employing a regular perturbation technique. The effect of chemical reaction and radiation absorption on all flow characteristics are discussed graphically.

KEYWORDS: Chemical reaction, radiation absorption, viscous fluid, porous medium and vertical channel.

1. INTRODUCTION

 Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. The effect of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and a knowledge of radiaton heat transfer becomes very important for the design pertinent equiptment. Nuclear power plants, gas turbines and the various propulsion devices foraircract, missiles, satellites and space vehicles are examples of such engineering areas.

Muthucumaraswamy and Ganesan [5] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka et al [2] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamka [1] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The effect of foreign mass on the free-convection flow past a semi-infinite vertical plate were studied [3]. Raptis and Perdikis [6] studied the unsteady free convection flow of water near 4 C in the laminar boundary layer over a vertical moving porous plate. Recently Ibrahim [4] studied the effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction.

2.FORMULATION OF THE PROBLEM

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We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls . The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at $y = L$ while the boundary at $y = L$ -L is maintained at constant temperature T1 while both the walls are maintained at uniform concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. We choose a rectangular Cartesian system $0 \times y$) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at $y=\pm L$.

The equations governing the unsteady flow, heat and mass transfer in terms of stream function ψ .

$$
[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = i \nabla^4 \psi - \beta g (T - T_0)_y -
$$

$$
- \beta^* g (C - C_0)_y - (\frac{\nu}{k}) \nabla^2 \psi
$$

(2.1)

$$
\rho_e C_p (\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}) = \lambda \nabla^2 \theta - Q (T - T_o) + Q_1 (C - C_o)
$$

(2.2)

$$
\left(\frac{\partial\phi}{\partial t} + \frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right) = D\nabla^2\phi - k_1(C - C_o)
$$
\n(2.3)

The boundary conditions for the velocity and temperature fields are

$$
\frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad T = T_1, \quad C = C_1 \quad \text{on } y = -L
$$

$$
\frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad T = T_2 + \Delta T_e \quad \text{Sin}(mx + nt), \quad C = C_2 \quad \text{on } y = L
$$

(2.4)

Introducing the non-dimensional variables as

$$
x' = mx, y' = y/L, t' = tvm2, \Psi' = \Psi / \nu, \theta = \frac{T - T_2}{T_1 - T_2}, \phi = \frac{C - C_2}{C_1 - C_2}
$$
\n(2.5)

the governing equations in the non-dimensional form (after dropping the dashes) are

$$
\delta R(\delta(\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)}) = \nabla_1^4 \psi + \left(\frac{G}{R}\right)(\theta_y + N\phi_y) - D^{-1}\nabla_1^2 \psi - M^2 \frac{\partial^2 \psi}{\partial y^2}
$$
\n(2.6)

$$
\delta P(\delta \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}) = \nabla_1^2 \theta - \alpha \theta + Q_2 \phi
$$

(2.7)

$$
\delta S c(\delta \frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial y}) = \nabla_1^2 \phi - \gamma \phi
$$

(2.8)

where

(2.10)

$$
R = \frac{UL}{V}
$$
 (Reynolds number)
\n
$$
P = \frac{\mu c_p}{k_1}
$$
 (Prandtl number),
\n
$$
Sc = \frac{V}{D_1}
$$
 (Schmidt number)
\n
$$
\alpha = \frac{QL^2}{\lambda}
$$
 (Schmidt number)
\n
$$
M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{V^2}
$$
 (Hartmann Number)
\n
$$
Q_2 = \frac{Q_1 (C_1 - C_2) L^2}{(T_1 - T_2)}
$$
 (Radiation
\nabsorption parameter)

1 2 $I_1 = \frac{H_1}{D}$ $\gamma_1 = \frac{K_1 L^2}{R}$ (Chemical reaction parameter) $\delta = mL$ (Aspect ratio) *n*

$$
\gamma = \frac{n}{vm^2}
$$
 (non-dimensional thermal wave velocity)

$$
\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}
$$

The corresponding boundary conditions are

$$
\psi(+1) - \psi(-1) = -1
$$

\n
$$
\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad at \ y = \pm 1
$$

\n(2.9)
\n
$$
\theta(x, y) = 1, \quad C = 1 \quad on \ y = -1
$$

\n
$$
\theta(x, y) = \sin(x + \pi), \quad C = 0 \quad on \quad y = +1
$$

\n
$$
\frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \quad at \quad y = 0
$$

\n
$$
\theta(x, y) = \sin(x + \pi), \quad C = 1 \quad \theta(x, y) = \sin(x + \pi), \quad C = 1
$$

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The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hyphothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

3. METHOD OF SOLUTION

 The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

We adopt the perturbation scheme and write

$$
\psi(x, y, t) = \psi_0(x, y, t) + \delta \psi_1(x, y, t) + \delta^2 \psi_{2}(x, y, t) + \dots
$$

\n
$$
\theta(x, y, t) = \theta_0(x, y, t) + \delta \theta_1(x, y, t) + \delta^2 \theta_2(x, y, t) + \dots
$$

\n
$$
\phi(x, y, t) = \phi_0(x, y, t) + \delta \phi_1(x, y, t) + \delta^2 \phi_2(x, y, t) + \dots
$$

\n(3.1)

On substituting (3.1) in (2.15) - (2.17) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$
\psi_{0, y y y y y} - M_1^2 \psi_{0, y y} = -\frac{G}{R} (\theta_{0, y} + NC_{0, y})
$$

(3.2)

$$
\theta_{0, yy} - \alpha \theta_0 + Q_2 \phi_0 = 0
$$

(3.3)

$$
\phi_{0, yy} - \gamma_1 \phi_0 = 0
$$

(3.4)

with

$$
\psi_0(+1)-\Psi_0(-1) = -1,
$$

\n
$$
\psi_{0,y} = 0, \psi_{0,x} = 0 \qquad \text{at } y = \pm 1
$$

\n(3.5)
\n
$$
\theta_o = 1, \qquad \phi_0 = 1 \qquad on \qquad y = -1
$$

\n
$$
\theta_o = Sin(x + \pi), \phi_0 = 0 \qquad on \qquad y = 1
$$

\n(3.6)

The first order are

$$
\psi_{1, y y y y y} - M_1^2 \psi_{1, y y} = -\frac{G}{R} (\theta_{1, y} + N \phi_{1, y}) + R(\psi_{0, y} \psi_{0, x y y} - \psi_{0, x} \psi_{0, y y y})
$$

(3.7)

$$
\theta_{1,yy} - \alpha \theta_0 = P(\psi_{0,x} \theta_{0,y} - \psi_{0,y} \theta_{0,x}) - Q_2 \phi_1
$$

(3.8)

$$
\phi_{1,yy} - \gamma_1 \phi_0 = Sc(\psi_{0,x} \phi_{0,y} - \psi_{0,y} \phi_{0,x})
$$

(3.9)

with

 $\Psi_{1(+1)}$ - $\Psi_{1(-1)} = 0$ $\psi_{1, y} = 0$, $\psi_{1, x} = 0$ at $y = \pm 1$ (3.10) $\theta_1(\pm 1) = 0$ $\phi_1(\pm 1) = 0$ at y = ± 1

The equations to the second order are

$$
\psi_{2, y y y y y} - M_1^2 \psi_{2, y y} = -\frac{G}{R} (\theta_{2y} + N \phi_{2,y}) + R(\psi_{0, y y} + \psi_{0, x} \psi_{1, y y y} +
$$

+
$$
\psi_{1, x} \psi_{0, y y y} - \psi_{0 y} \psi_{1, xy} - \psi_{1, y} \psi_{0, xy}
$$

(3.11)

$$
\theta_{21,y} - \alpha \theta_2 = P(\theta_{o,t} + \psi_{0,x}\theta_{1,y} - \psi_{0,y}\theta_{1x} - \psi_{1,y}\theta_{o,x} + \psi_{1,x}\theta_{o,y}) - Q_2\phi_2
$$
\n(3.12)
$$
\phi_{2,yy} - \gamma_1 \phi_2 = Sc(\phi_{o,t} + \psi_{0,x}\phi_{1,y} - \psi_{0,y}\phi_{1x} - \psi_{1,y}\phi_{o,x} + \psi_{1,x}\phi_{o,y})
$$
\n(3.13)

with

$$
\psi_{2(+1)}.\psi_{2(-1)} = 0
$$

\n
$$
\psi_{2, y} = 0, \psi_{2, x} = 0 \qquad \text{at } y = \pm 1
$$

\n(3.14)
\n
$$
\theta_2(\pm 1) = 0 \qquad \phi_2(\pm 1) = 0 \qquad \text{at } y = \pm 1
$$

\n(3.18)

The equations $3.2 - 3.4 \& 3.7 - 3.9 \& 3.11 - 3.13$ are solved subject to the conditions 3.5, 3.6, 3.10 & 3.14 to obtain the expressions for velocity, temperature & concentration.

4. NUSSELT NUMBER and SHERWOOD NUMBER

Knowing the temperature & concentration the local rate of heat and mass transfer on the walls have been calculated using the formula

$$
Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y}\right)_{y=\pm 1}
$$

where $\theta_m = 0.5 \int_{-1}^{1} \theta \, dy$

and
$$
Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y}\right)_{y=\pm 1}
$$

where

$$
= 0.5 \int_C dy
$$

where ¹⁴²¹,..........,.......... *ddd* are constants.

 $C_{\scriptscriptstyle m}$

1

Fig.1 represents the variation of u with radiation absorption co-efficient Q1. An increase in $Q1 \leq 1.5$ leads to an enhancement |u| and depreciates at higher $Q1 = 2$ and again enhances at still higher value of $Q1 = 3.5$. The effect of chemical reaction on u is shown in fig.2. We find that the velocity exhibits the reversal flow which appears in the entire region at $\gamma=1$ disappears everywhere in the region with higher γ >0 (degenerating chemical reaction case) No such phenomena is observed for γ <0. Also |u| enhances with γ \le 2.5 and depreciates with higher $\gamma \leq 3.5$, while the variation of u with $\gamma \leq 0$ (generating chemical reaction) shows that |u| depreciates with increase in $|\gamma|$. From fig.3 we infer that $|u|$ enhances with increase in x+ γt $\leq \pi/2$ and depreciates at x+yt= π again enhances at higher x+y= 2π . From fig.4, the variation of v with radiation absorption parameter Q1 reveals that the secondary velocity enhances with increase in $O1 \le 1.3$ and depreciates at $O1 = 2$ and again enhances at higher value of $O1 = 3.5$. The variation of v with chemical reaction parameter γ shows that for $\gamma \le 1.5$ it is towards the midregion and is towards the boundary for higher $\gamma \geq 2.5$.

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Also enhances with $\gamma \leq 2.5$ and depreciates with higher $\gamma \geq 3.5$, which it depreciates with $|\gamma|$ everywhere in the region(fig.5). The variation of v with the phase $x+y$ of the boundary temperature exhibits that the secondary velocity is towards the boundary for smaller and higher values of $x+y$ and is towards midregion for intermediate values of $x+y$. Also |v| enhances with increase in $x+y \leq \pi$ and depreciates with higher $x+y \geq 2\pi$. The effect of radiation absorption on θ is shown in fig.7. It is found that an increase in $O1 \le 1$ enhances the temperature, depreciates with higher $Q1=1.5$ and for still higher $Q1=3.5$ leads to an enhancement in θ . Thus the inclusion of radiation absorption in the Energy equation helps to enhance the heat transfer in the flow region. The variation of θ with chemical reaction parameter γ shows that an increase in $\gamma \leq 2.5$ leads to an enhancement in θ while for higher $\gamma \geq 3.5$ results a depreciation in θ . While an increase in $\gamma \leq 0$ (generating chemical reaction case) smaller the actual temperature in the flow region(fg.8).

Fig.9 illustrates that an increase in phase $x+y \leq \pi/2$ enhances the temperature and for higher $x+yt=\pi$ it depreciates and for still higher $x+yt=2\pi$ the temperature experiences an enhancement. The variation of C with chemical reaction parameter γ for smaller and higher

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values of the $\gamma(\gamma=1 \& 3.5)$ the concentration is positive in the entire flow region while for intermediate values of γ the concentration changes from positive to negative as we move from left half to right half. For an increase in $\gamma \leq 1.5$ enhances the actual concentration and for higher $\gamma = 2.5$ smaller the actual concentration in actual concentration. While an increase n $\gamma \le 0$ enhances the actual concentration in the entire flow region (fig.10). Fig.11 illustrates the variation of C with phase $x+yt$. We found that an increase in $x+yt\leq \pi/2$ depreciates the actual concentration and enhances with $x+yt=\pi$ and for still higher $x+yt=2\pi$ the actual concentration depreciates in the entire flow region

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Table -2 : Nusselt number (Nu) at $y = +1$

The rate of heat transfer at the boundaries at $y = \pm 1$ is shown in tables 1-2 for different values of, Sc, Q1, γ , and N. The variation of Nu with Schmidt number Sc shows that lesser the molecular diffusivity larger Nu at $y = -1$ and at $y = +1$ smaller |Nu| and for further lowering of the molecular diffusivity higher |Nu|. The variation of Nu with radiation absorption coefficient Q1 and chemical reaction parameter γ shows that the an increase in Q1 or γ results in an enhancement in Nu for all G at both the walls (Tables 1&2).

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The variation Nu with buoyancy ratio N shows that when molecular buoyancy force dominates over thermal buoyancy force the rate of heat transfer depreciates when buoyancy forces act in the same direction and enhances when they act in opposite direction at both the walls (tables 1 & 2). The Sherwood Number (Sh) at $y = \pm 1$ is exhibited in tables 3-4 for a different variations. The variation of Sh with Sc shows that lesser the molecular diffusivity larger $|\text{Sh}|$ at y = -1 while at y = +1 larger $|\text{Sh}|$ for G>0 and smaller $|\text{Sh}|$ for G<0 (tables 3 & 4). The variation of Sh with Q1 and γ exhibits that the rate of mass transfer enhances with increase in radiation absorption parameter Q1 at $y = \pm 1$. An increase in chemical reaction parameter $\gamma \leq 4$ depreciates Sh and enhances with higher $\gamma \geq 6$ for G > 0 and for G ≤ 0 it enhances with γ while at y = +1 larger $|Sh|$ (tables 3 & 4). The variation of Sh with buoyancy ratio N shows that when molecular buoyancy force dominates over thermal buoyancy forces act in the same direction while for forces acting in the opposite direction it enhances at both the walls. (Tables 3& 4).

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