A STUDY OF UNSTEADY FREE CONVECTIVE FLOW OF MHD WITH CONSTANT SUCTION POROUS MEDIA

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Abstract: we analyzed the nonlinear unsteady MHD flow of viscous, incompressible and electrically conducting fluid past a vertical porous channel under theinfluence of thermal radiation and chemical reaction. We observed how various parameters affect the flow past an infinite vertical accelerated plate.

Introduction:

Viscoelastic flows arise in numerous processes in chemical engineering systems. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects. An extensive range of mathematical models has been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. An eloquent exposition of viscoelastic fluid models has been presented by Joseph. Examples of such models are the Oldroyd model, Johnson–Seagalman model, the upper convected Maxwell model, and the Walters-B model. Both steady and unsteady flows have been investigated at length in a diverse range of geometries using a wide spectrum of analytical and computational methods. Siddappa and Khapate studied the second order Rivlin–Ericksen viscoelastic boundary layer flow along a stretching surface. Rochelle and Peddieson used an implicit difference scheme to analyze the steady boundary-layer flow of a nonlinear Maxwell viscoelastic fluid past a parabola and a paraboloid. Ji et al. studied the Von Karman Oldroyd-B viscoelastic flow from a rotating disk using the Galerkin method with B-spline test functions.

Consider unsteady hydromagnitic free convective flow of incompressible and electrically conducting fluid past an infinite vertical porous plate in the presence of constant suction and heat absorbing sink through porous media. Let the x-axis be taken in the vertically upward direction along the infinite vertical plate and y-axis normal to it. Boussineq's approximation, for the equations of the flow is governed as:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \sigma\frac{B_0^2}{\rho}u' - \nu\frac{u'}{K^*} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{3}$$

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} - K_r'^2(C - C_\infty) \tag{4}$$

The boundary condition for the velocity, temperature and concentration fields are :

$$u' = U_0, T = T_w + \varepsilon(T_w - T_\infty)e^{n't'}$$

$$C = C_w + \varepsilon(C_w - C_\infty)e^{n't'} \text{ at } y' = 0$$

$$u' \to U^*(t'), T \to T_\infty, C \to C_\infty \text{ at } y' \to \infty \tag{5}$$

By using Rosseland approximation, the radiative heat flux is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y'} \tag{6}$$

If the temperature difference T - T_∞ within the flow is sufficiently small, the Taylor series for T^4 neglecting higher order terms is given by a linear temperature function :

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

In view of the equations (6) and (7), equation (3) reduces to :

$$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \alpha\frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma_s T_\infty^3}{3k_e \rho c_p}\frac{\partial^2 T}{\partial y'^2} + \frac{v}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{8}$$

From the continuity Eq. (1), it is clear that the suction velocity normal to the plate which can be considered either a constant or a function of time. In the fitness of the present situation, it is assumed in the form:

$$v' = -V_0\left(1 + \varepsilon e^{n't'}\right) \tag{9}$$

the negative sign indicates that the suction is towards the plate.

In order to write the governing equations and boundary conditions in non-dimensional form, the following non-dimensional scheme is introduced.

$$y = \frac{y'V_0}{v}, t = \frac{t'V_0^2}{v}, u = \frac{u'}{U_0}, v = \frac{v}{V_0}, U(t) = \frac{U^*(t')}{U_0},$$

$$n = \frac{n'V}{V_0^2}, G_r = \frac{vg\beta(T_w - T_\infty)}{U_0 V_0^2}, G_m = \frac{vg\beta^*(C_w - C_\infty)}{U_0 V_0^2},$$

$$M = \frac{\sigma\beta_0^2 v}{U_0^2}, R = \frac{16\sigma_s T_\infty^3}{3k_e k}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\varphi = \frac{C - C_\infty}{C_w - C_\infty}, P_r = \frac{v}{\alpha}, S_c = \frac{v}{D}, E_c = \frac{U_0^2}{c_p(T_w - T_\infty)},$$

$$K = \frac{K^* V_0^2}{v^2}, K_r^2 = \frac{K_r'^2 v}{V_0^2} \tag{10}$$

By using the Eqs. (9) and (10), Eqs (2), (8) and (4) are reduced to the following dimensionless form:

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon e^{n't'}\right)\frac{\partial u}{\partial t} = G_r\theta + G_m\varphi + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \tag{11}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon e^{n't'}\right)\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

$$\frac{\partial \varphi}{\partial t} - \left(1 + \varepsilon e^{n't'}\right)\frac{\partial \varphi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \varphi}{\partial y^2} - K_r^2\varphi \tag{13}$$

The corresponding dimensionless boundary conditions are :

u = 1, θ = 1 + εe^{nt}, φ = 1 + εe^{nt} at y = 0

$u \to U(t),\ \theta \to 0,\ \varphi \to 0$ at $y \to \infty$ (14)

Non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as :

$u(y, t) = u_0(y) + \varepsilon e^{nt} u_1(y)$

$\theta(y, t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y)$

$\varphi(y, t) = \varphi_0(y) + \varepsilon e^{nt} \varphi_1(y)$

also

$U(t) = (1 + \varepsilon e^{nt})$ (15)

Substituting the equation (15) in the Eqs (11-13), we obtain the following equations by considering harmonic and non-harmonic terms while neglecting the higher terms with order of $0(\varepsilon)^2$

$$u_0'' + u_0' - N_1 u_0 = -G_r \theta_0 - G_m \varphi_0 \quad (16)$$

$$u_1'' + u_1' - N_2 u_1 = -G_r \theta_1 - G_m \varphi_1 \quad (17)$$

$$\theta_0'' + N_3 \theta_0' = E_c N_2 \left(\frac{\partial u_0}{\partial y} \right)^2 \quad (18)$$

$$\theta_1'' + N_3 \theta_1' - N_4 \theta_1' = N_3 \theta_0' - 2 N_3 E_c u_0' u_1' \quad (19)$$

$$\varphi_1'' + S_c \varphi_1' - K_r^2 S_c \varphi_0 = 0 \quad (20)$$

$$\varphi_1'' + S_c \varphi_1' - (K_r^2 + \omega) S_c \varphi_1 = S_c \varphi_0' \quad (21)$$

Subject to the boundary conditions :

$u_0 = 1,\ \theta_0 = 1,\ \varphi_0 = 1,\ u_1 = 0,\ \theta_1 = 1,\ \varphi_1 = 1$ at $y = 0$

$u_0 \to 1,\ \theta_0 \to 0,\ \varphi_0 \to 0,\ u_1 \to 1,\ \theta_1 \to 0,\ \varphi_1 \to 0$ at $y = \infty$ (22)

To solve the non linear-coupled Eqs. (16-21), we further assume that the viscous dissipation parameter (Eckert number E_c) is very small for incompressible flows, and therefore, advance an asymptotic expansion for the flow velocity, temperature and concentration as follows:

$u_0(y) = u_{01}(y) + E_c u_{02}(y)$

$\theta_0(y) = \theta_{01}(y) + E_c \theta_{02}(y)$

$\varphi_0(y) = \varphi_{01}(y) + E_c \varphi_{02}(y)$

$u_1(y) = u_{11}(y) + E_c u_{12}(y)$

$\theta_1(y) = \theta_{11}(y) + E_c \theta_{12}(y)$

$\varphi_1(y) = \varphi_{11}(y) + E_c \varphi_{12}(y)$ (23)

Substituting equation (23) into equations (16-21), we obtain the following sequence of approximations :

The zeroth order equations are :

$$u_{01}'' + u_{01}' - N_1 u_{01} = -G_r \theta_{01} - G_m \varphi_{01} \quad (24)$$

$$u_{02}'' + u_{02}' - N_1 u_{02} = -G_r \theta_{02} - G_m \varphi_{02} \quad (25)$$

$$\theta_{01}'' + N_3 \theta_{01}' = 0 \quad (26)$$

$$\theta_{02}'' + N_3\theta_{02}' = -N_3\left(\frac{\partial u_{01}}{\partial y}\right)^2 \tag{27}$$

$$\varphi_{01}'' + S_c\varphi_{01}' - K_r^2 S_c\varphi_{01} = 0 \tag{28}$$

$$\varphi_{02}'' + S_c\varphi_{02}' - K_r^2 S_c\varphi_{02} = 0 \tag{29}$$

Subject to the boundary conditions :

$u_{01} = 1,\ u_{02} = 0,\ \theta_{01} = 1,\ \theta_{02} = 0,\ \varphi_{01} = 1,\ \varphi_{02} = 0$ on $y = 0$

$u_{01} \to 1,\ u_{02} \to 0,\ \theta_{01} \to 1,\ \theta_{02} \to 0,\ \varphi_{01} \to 1,\ \varphi_{02} \to 0$ at $y \to \infty$ (30)

The first order equations are :

$$u_{11}'' + u_{11}' - N_2 u_{11} = -G_r\theta_{11} - G_m\varphi_{11} \tag{31}$$

$$u_{12}'' + u_{12}' - N_1 u_{12} = -G_r\theta_{12} - G_m\varphi_{12} \tag{32}$$

$$\theta_{11}'' + N_3\theta_{11}' - N_4\theta_{11}' = 0 \tag{33}$$

$$\theta_{12}'' + N_3\theta_{12}' - N_4\theta_{12}' = -N_3\theta_{02}' - 2N_3 u_{01}' u_{11}' \tag{34}$$

$$\varphi_{11}'' + S_c\varphi_{11}' - (K_r^2 + n)S_c\varphi_{11} = -S_c\varphi_{01}' \tag{35}$$

$$\varphi_{12}'' + S_c\varphi_{12}' - (K_r^2 + n)S_c\varphi_{12} = S_c\varphi_{02}' \tag{36}$$

where

$$N_1 = M + \frac{1}{K},\ N_2 = M + \frac{1}{K} + n,$$

$$N_3 = \frac{3RP_r}{3R+4},\ N_4 = \frac{3RP_r n}{3R+4}$$

Subject to the boundary conditions :

$u_{11} = 0,\ u_{12} = 0,\ \theta_{11} = 1,\ \theta_{12} = 0,\ \varphi_{11} = 1,\ \varphi_{12} = 0$ on $y = 0$

$u_{11} \to 1,\ u_{12} \to 0,\ \theta_{11} \to 0,\ \theta_{12} \to 0,\ \varphi_{11} \to 0,\ \varphi_{12} \to 0$ at $y \to \infty$ (37)

Solving Eqs. (24-29) under the boundary conditions in Eq. (30) and Eqs. (31-36) under the boundary conditions in Eq.(37), and using Eqs. (15) and (23), we obtain the velocity temperature and concentration distributions in the boundary layer as :

$$\begin{aligned}
u(y,t) = {} & A_6 e^{-m_4 y} + 1 - A_4 e^{-N_3 y} - A_5 e^{-m_1 y} + E_c\{A_{80} e^{-m_4 y} \\
& - A_{73} e^{-m_3 y} + A_{74} e^{-2m_4 y} + A_{75} e^{-2N_3 y} + A_{76} e^{-2m_1 y} \\
& - A_{77} e^{-(m_4+N_3)y} + A_{78} e^{-(m_1+N_3)y} - A_{79} e^{-(m_4+m_1)y}\} \\
& + \varepsilon e^{\omega t}[\{A_{12} e^{-m_5 y} + 1 + A_7 e^{-m_3 y} - A_8 e^{-N_3 y} \\
& - A_9 e^{-m_2 y} - A_{10} e^{-m_1 y} + A_{11} e^{-m_2 y}\} \\
& + E_c\{A_{72} e^{-m_5 y} - A_{46} e^{-m_3 y} - A_{47} e^{-N_3 y} \\
& + A_{48} e^{-2m_4 y} + A_{49} e^{-2N_3 y} + A_{50} e^{-2m_1 y} \\
& - A_{51} e^{-(m_4+N_3)y} + A_{52} e^{-(m_1+N_3)y} - A_{53} e^{-(m_4+m_1)y} \\
& + A_{54} e^{-(m_4+m_5)y} - A_{58} e^{-(N_3+m_3)y} - A_{59} e^{-(m_1+m_3)y} \\
& + A_{60} e^{-(m_4+N_3)y} - A_{61} e^{-2N_3 y} - A_{62} e^{-(m_1+N_3)y} \\
& + A_{63} e^{-(m_4+m_2)y} - A_{64} e^{-(m_2+N_3)y} + A_{65} e^{-(m_1+m_2)y}
\end{aligned}$$

$$-A_{66}e^{-(m_1+m_4)y}-A_{67}e^{-(m_1+N_3)y}\}]$$

$$\theta(y,t)=e^{-N_3y}+E_c\{A_{19}e^{-N_3y}-A_{13}e^{-2m_4y}-A_{14}e^{-2N_3y}$$
$$-A_{15}e^{-2m_1y}+A_{16}e^{-(m_4+N_3)y}-A_{17}e^{-(m_1+N_3)y}$$
$$-A_{18}e^{-(m_1+m_4)y}\}+\varepsilon e^{\omega t}[\{A_3e^{-m_3y}-A_2e^{-N_3y}\}$$
$$+E_c\{A_{45}e^{-m_3y}+A_{20}e^{-N_3y}-A_{21}e^{-2m_4y}$$
$$-A_{22}e^{-N_3y}-A_{23}e^{-2m_1y}+A_{24}e^{-(m_4+N_3)y}$$
$$-A_{25}e^{-(m_1+N_3)y}+A_{26}e^{-(m_4+m_1)y}-A_{27}e^{-(m_4+m_5)y}$$
$$+A_{28}e^{-(N_3+m_5)y}+A_{29}e^{-(m_1+m_5)y}-A_{30}e^{-(m_4+m_3)y}$$
$$+A_{31}e^{-(N_3+m_3)y}+A_{32}e^{-(m_1+m_3)y}-A_{33}e^{-(m_4+N_3)y}$$
$$-A_{34}e^{-2N_3y}+A_{35}e^{-(m_1+N_3)y}-A_{36}e^{-(m_4+m_2)y}$$
$$-A_{40}e^{-(m_1+N_3)y}+A_{41}e^{-2m_1y}-A_{42}e^{-(m_4+m_2)y}$$
$$+A_{43}e^{-(m_2+N_3)y}-A_{44}e^{-(m_1+m_2)y}\}]$$

$$\varphi(y,t)=e^{-m_1y}+\varepsilon e^{\omega t}[e^{-m_2y}+A_1(e^{-m_2y}-e^{-m_2y})]$$

Here, we examine the nature of velocity and temperature profiles for different values of various physical parameters associated with the problem under consideration. Eqs (24-29) with the help of Eq.(30) and the Eqs (3-36) with the help of Eq.(37) are solved numerically in PDE Solver. The velocity and temperature profiles are obtained for various parameters like magnetic parameters, porosity parameter, free convection parameter, chemical reaction parameter, Schmidt number and Eckert number, frequency of excitation and radiation parameter, while the values of some physical parameters are fixed as real constant ε = 0.1, Prandt1 number $P_r = 0.71$ to represent air and time parameter t = 0.75.

Result and Discussion:

1 Effect of magnetic parameters on velocity and temperature profiles

In figs (1) and (2), temperature and velocity profiles are shown for different values of M. It does not have too much effect on temperature and velocity profiles.

2 Effect of porosity parameter on velocity and temperature profiles

Figures (3 and 4) show the effect of porosity parameter K on temperature profiles and velocity profiles. As K increases, considerable enhancement is observed in temperature and velocity profiles, respectively. It is seen that porosity parameter has strong effect on velocity profiles in comparison to temperature profiles. After displacement value (y = 25), the temperature variations are negligible and velocity starts fluctuating for all values of K.

3 Effect of Grashof number, chemical reaction parameter, Schmidt number and Eckert number on temperature profiles

From Figs (5), (7) (9) and (11), we can see that Grashof number G_r, chemical reaction parameter K_r, Schmidt number S_c, Eckert number E_c, don't have appreciable effects on temperature profiles. It is approximately converging to the value of 0.039327 at displacement y = 21, for all values of parameters.

4 Effect of Grashof number on velocity profiles

Figure 6 shows velocity variations with G_r (thermal Grashof number) in cases of heating of the surface. It is found that the velocity profiles increase with increase of Grashof number. Finally, it is found that in the case of heating of plate, near the surface of the plate, the velocity profiles increase and become maximization and then decrease on moving away from the plate.

5 Effect of chemical reaction parameter on velocity profiles

Figure 8 shows that near the surface of the plate, velocity is strongly increasing and reaches its maximum value and then decreases with respect to displacement y, however, the destructive chemical reaction ($K_r > 0$) has not too much effect on velocity profiles.

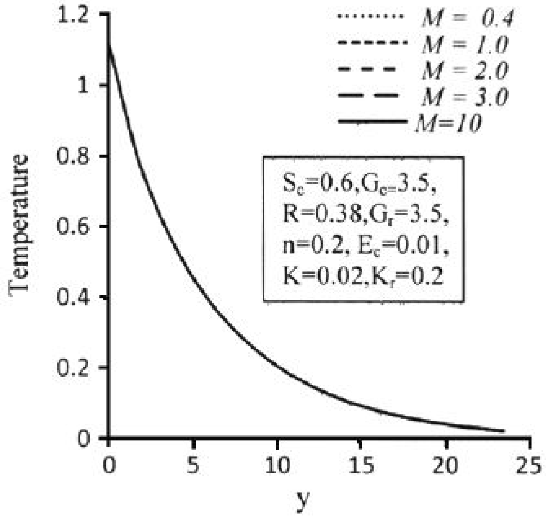

Fig. 1 – Effect of magnetic parameter on temperature field

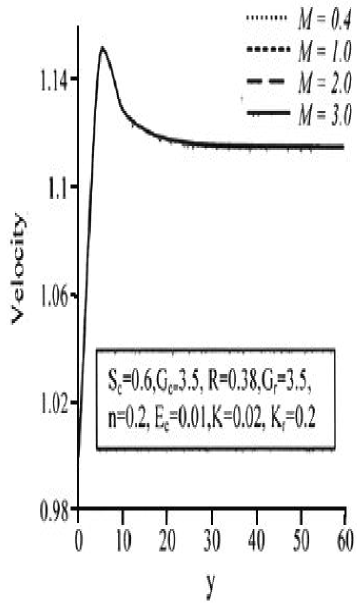

Fig.2 – Effect of magnetic parameter on velocity field

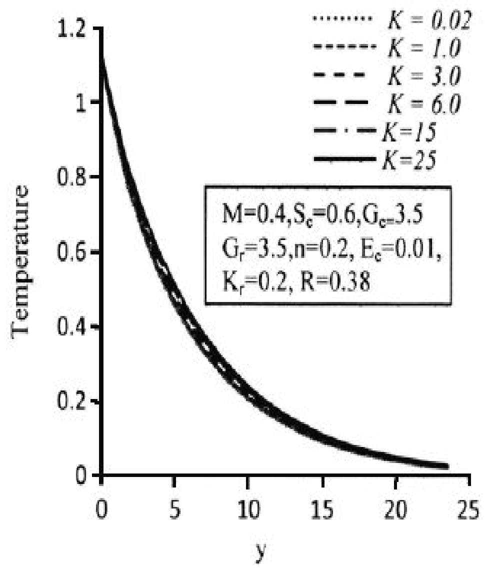

Fig. 3 – Contribution of porosity parameter on temperature field

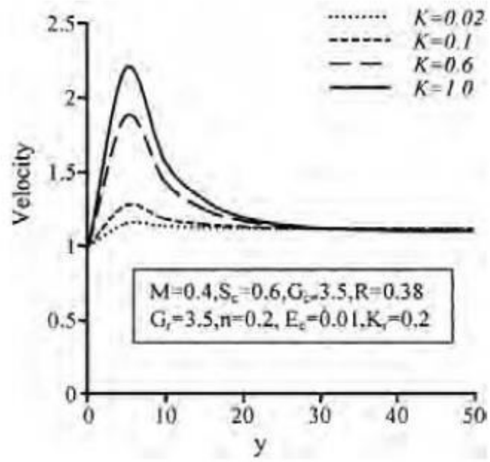

Fig. 4 – Contribution of porosity parameter on velocity field

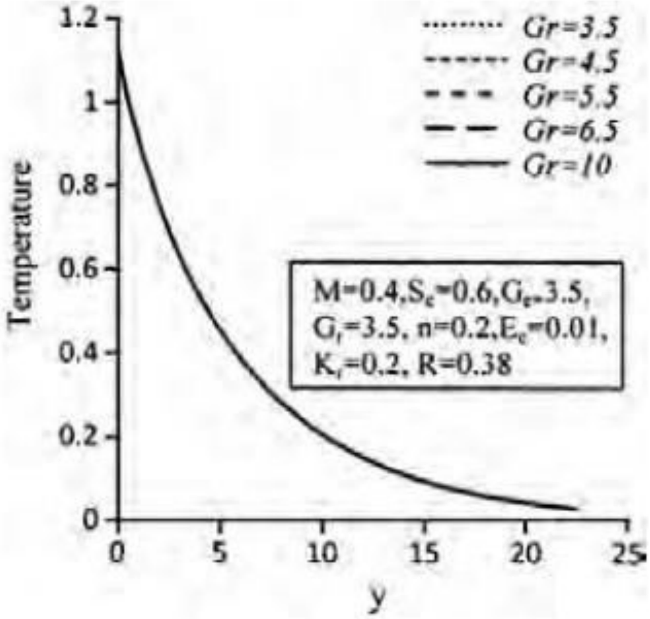

Fig. 5 – Effect of free convection parameter on temperature field

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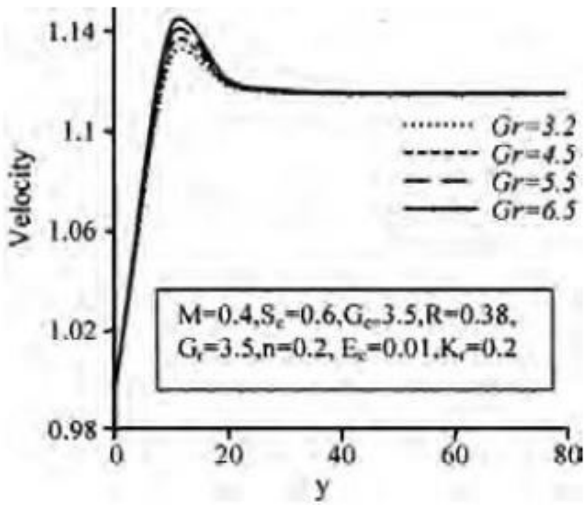

Fig. 6 – Effect of free convection parameter on velocity field

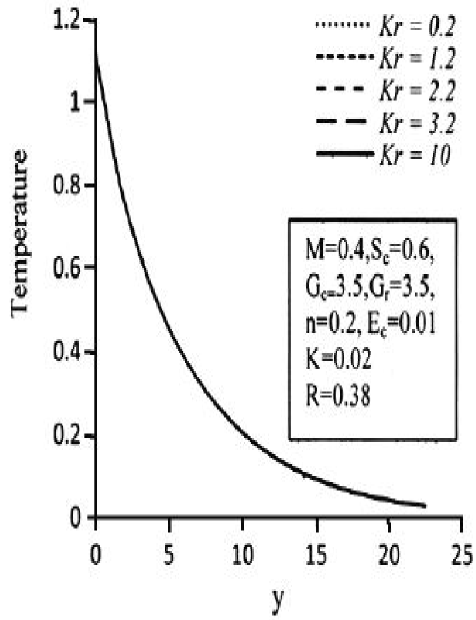

Fig. 7 – Effect of chemical reaction parameter on temperature field

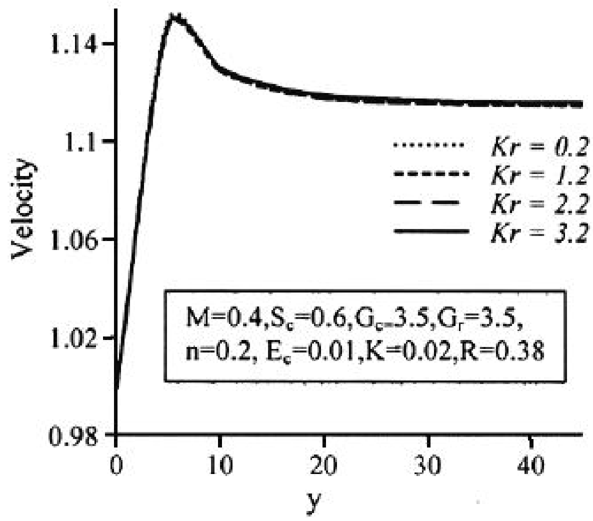

Fig. 8 – Effect of Chemical reaction parameter on velocity field

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