

A STUDY OF SEMI PRIME IDEALS OF AN Γ-SEMIGROUPS.

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ABSTRACT: In this paper we introduce the concept of weakly prime ideals and semi prime ideals and gave some characterizations of an ordered commutative Γ-semigroup. In this paper we characterized the properties of non-empty subset of a partially ordered commutative Γ-semigroup.

KEYWORDS: Commutative Γ-semigroups, partially ordered commutative Γ-semigroup, Weakly Prime Ideals and semi prime ideals.

INTRODUCTION AND PRELIMINARIES

Young In Kwon and Sang Keun Lee [1] have introduced the concept of weakly prime-ideals in po- Γ- Semi groups and gave some characterizations of weakly prime ideals. In [2] they have introduced the concepts of weakly prime and weakly semi-prime ideals in po- Γ- Semi groups and obtained the results on characterizations of weakly prime and weakly semi-prime ideals of po-Γ-Semi groups analogous to the results given by N.Kehayopulu on characterizations of weakly prime and weakly semi-prime ideals of po-Semi groups. In [3] they gave some properties of special elements in po-Γ-Semi group. M.Y Abbasi and abul Basar [4] have defined the involution in po-Γ-Semi group and extended the results of prime, semi-prime and weakly prime ideals to the involution po-Γ-Semi group. They gave characterizations of intra-regular involution po-Γ-Semi groups and establish that in involution po-Γ-Semi group the involution preserves the order. Finally proved that the ideals of a po-Γ-Semi group with order preserving involution are prime if and only if it is intra-regular.

Definition 1:-A non-empty subset A of a partially or ordered commutative Γ-semigroup M is an ideal of M if

i) $A\,\Gamma M \subseteq A$ or $M\,\Gamma A \subseteq A$ ii) $a \in A, b \leq a$ for some $b \in M \Rightarrow b \in A$

Definition 2:-A non-empty subset T of a partially ordered commutative Γ-semigroup M is weakly prime if $A\Gamma B \subseteq T \Rightarrow A \subseteq T$ or $B \subseteq T$ for any two ideals A and B of M.

Definition 3:-A non-empty subset T of a partially ordered commutative Γ-semigroup M is weakly semi-prime if $A\Gamma A \subseteq T \Rightarrow A \subseteq T$ for any ideal A of M.

Notation:-for $V \subseteq M$, $(V] = \{a \in M: a \leq v \text{ for some } v \in V\}$.We write $(a]$ instead of $(\{a\}]$.

WEAKLY SEMI PRIME IDEALS

Theorem 1:- An ideal T of a partially ordered commutative Γ-semi group M is prime if and only if T is both weakly prime and semi prime.

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Proof: Suppose the ideal T is prime ideal of M.Then it is obvious that T is weakly prime and semi prime. On the other hand if T is both weakly prime and semi prime then proof is exactly similar to that of theorem 2.8 ii) ⇒ iii) case.

Theorem 2:-Let M be a partially ordered commutative Γ-semigroup and T is an ideal of M then the following are equivalent.

i). T is weakly semi prime.

ii). $(a\Gamma M\Gamma a] \subseteq T$ for all $a \in M \Rightarrow a \in T$.

iii). $I(a)\Gamma I(a) \subseteq T$ for all $a \in M \Rightarrow a \in T$.

iv).For every ideal A of M such that $A\Gamma A \subseteq T \Rightarrow A \subseteq T$

Proof:-i) ⇒ ii).

Let T is weakly semi-prime. Let $a \in M$, $(a\Gamma M\Gamma a] \subseteq T$.

Consider,$(M\Gamma a\Gamma M]\Gamma(M\Gamma a\Gamma M] \subseteq ((M\Gamma a\Gamma M)\Gamma (M\Gamma a\Gamma M)]$

$$\subseteq (M\Gamma(a\Gamma M\Gamma a)\Gamma M]$$

$$\subseteq (M\Gamma T\Gamma M]$$

$$\subseteq T = T.$$

Since $(M\Gamma a\Gamma M]$ is an ideal of M and T is weakly semi-prime we have $(M\Gamma a\Gamma M] \subseteq T$.

Then we get $(I(a)\ \Gamma\ I(a)]\ \Gamma\ (I(a)\ \Gamma\ I(a)] \subseteq (T] = T$.

As T is weakly semi-prime $(I(a)\ \Gamma\ I(a)]$ is an ideal of M,we have

$(I(a)\ \Gamma\ I(a)] \subseteq T$

$\Rightarrow I(a)\ \Gamma\ I(a) \subseteq T$ and $I(a)$ is an ideal of M,we have $I(a) \subseteq T$.

Hence $a \in I(a) \subseteq T \Rightarrow a \in T$.

ii) ⇒ iii)

Let $I(a)\ \Gamma\ I(a) \subseteq T$.

$a \in I(a) \Rightarrow (a] \subseteq (I(a)]$,

Also $M\Gamma a \subseteq a \cup M\Gamma a \cup M\Gamma a\Gamma M = I(a)$.

$\therefore (M\Gamma a] \subseteq (I(a)] \Rightarrow (a]\ \Gamma\ (M\Gamma a] \subseteq (I(a)]\ \Gamma\ (I(a)]$.

$\therefore a\Gamma M\Gamma a \subseteq (a]\ \Gamma\ (M\Gamma a] \subseteq (I(a)]\ \Gamma\ (I(a)]$

$$= I(a)\ \Gamma\ I(a) \subseteq T.$$

$$(a\Gamma M\Gamma a] \subseteq (T] = T \Rightarrow a \in T.$$

iii) ⇒ iv)

Let $A\Gamma A \subseteq T$ and $a \in A \therefore I(a) \subseteq (A \cup M\Gamma A]$.

$$I(a)\, \Gamma\, I(a) \subseteq (A \cup M\Gamma A]\, \Gamma\, (A \cup M\Gamma A]$$

$$\subseteq (A\Gamma A \cup A\Gamma M\Gamma A \cup M\Gamma A\Gamma A \cup M\Gamma A\Gamma M\Gamma A]$$

$$\subseteq (T \cup T \cup T \cup T]$$

$$\subseteq (T] = T.$$

$$\therefore I(a)\, \Gamma\, I(a) \subseteq T \Rightarrow a \in T \Rightarrow A \subseteq T.$$

iv) ⇒ i) is obvious.

Definition: An ordered commutative Γ –semigroup is called chain ordered commutative Γ-semigroup if Γ-ideals of M form a chain. i.e., for any Γ-ideals I and J of M, we have either $I \subseteq J$ or $J \subseteq I$.

Theorem: Let M be a chain ordered commutative Γ –semigroup. Then a Γ-ideal I of M is semi prime Γ-ideal if and only if I is prime Γ-ideal of M.

Proof: Assume that I is a semiprime Γ-ideal of M. Let A and B be any two two ideals sof M such that $A\Gamma B \subseteq I$. Since M is chain ordered commutative Γ-semigroup, we have $A \subseteq B$ or $B \subseteq A$. If $A \subseteq B$ then $A\Gamma A \subseteq A\Gamma B \subseteq I$ and $A \subseteq I$ follows. Similary if $B \subseteq A$, then $B \subseteq I$.Thus I is prime Γ-ideal of M. The converse is obvious.

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