ON β-OPEN SETS IN TOPOLOGICAL SPACES

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Abstract: In this paper, we have studied $\boldsymbol{\beta}$-Open sets in topological space. We have obtained some significant properties of $\boldsymbol{\beta}$-Open sets and constructed various examples.

Keywords: Topological spaces, interior, closure, open set, semi- closed set.

Definition 1.6.1[1]: Let (X, τ) be a topological space and let $A \subseteq X$. ThenA is said to be $\boldsymbol{\beta}$ **−open set** if $A \subseteq Cl\left(Int\left(Cl\left(A\right)\right)\right)$.

Remark1.6.1: In a topological space (X, τ) the empty set $\emptyset$ and whole set X are β −open sets.

Example1.6.1: Let $X = \{x_1, x_2, x_3\}$ be the topological space with respect to topology $\tau = \{\emptyset, X, \{x_1\}, \{x_1, x_3\}\}$ on X. Let us consider the set $A = \{x_1, x_2\}$ in X. We see that $Int\left(Cl\left(A\right)\right) = X$. This gives $Cl\left(Int\left(Cl\left(A\right)\right)\right) = X$.Therefore $A \subseteq Cl\left(Int\left(Cl\left(A\right)\right)\right)$ and hence A is aβ −open set in X.

Proposition 1.6.1: In a topological space (X, τ) each open set is $\boldsymbol{\beta}$–open but not the converse.

Proposition 1.6.2: In a topological space (X, τ) each α −open set is $\boldsymbol{\beta}$ –open but not the converse.

Proposition 1.6.3: In a topological space (X, τ) each semi-open set is $\boldsymbol{\beta}$ –open but not the converse.

Proposition 1.6.4: In a topological space (X, τ) each pre- open set is $\boldsymbol{\beta}$ –open but not the converse.

Theorem1.6.1 [1]: Let (X, τ) be a topological space and let A ⊆ X. Then A is$\boldsymbol{\beta}$ -open if and only if Cl (A) $= Cl\left(Int\left(Cl\left(A\right)\right)\right)$.

Proposition1.6.5 [1]: In a topological space(X, τ) arbitrary union of $\boldsymbol{\beta}$ -open sets is $\boldsymbol{\beta}$ -open.

Remark1.6.2: In a topological space(X, τ) intersection of two $\boldsymbol{\beta}$ -open sets may not be $\boldsymbol{\beta}$ -open. Thus the family of all $\boldsymbol{\beta}$ -open sets in (X, τ) need not a topology on X.

Theorem1.6.2 [1]: Let (X, τ) be a topological space and let A ⊆ X .Then A is$\boldsymbol{\beta}$ -open if and only if for each x∈ A there exists a $\boldsymbol{\beta}$ -open set U in X such that x ∈U⊆ A.

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Definition1.6.2 [7]: Let (X, τ) be a topological space and let$A \subseteq X$. Then union of all $\boldsymbol{\beta}$-open sets in X contained in A is called **$\boldsymbol{\beta}$-interior** of A. Theβ-interior of $A$is denoted by $\boldsymbol{\beta}i(A)$.

Remark1.6.3: We note that $\boldsymbol{\beta}i(A)$ is the largest $\boldsymbol{\beta}$ -open set in (X, τ) contained in A.

Proposition 1.6.6: Let(X, τ) be a topological space andlet$A \subseteq X$. Then A is$\boldsymbol{\beta}$ -open if and only if$\boldsymbol{\beta}i(A) = A$.

Proposition 1.6.7: Let(X, τ) be a topological space and let A, B be subsets of X. Then following properties holds:

(i) $\beta i(\phi) = \phi, \beta i(X) = X$.
(ii) If $A \subseteq B$then$\beta i(A) \subseteq \beta i(B)$.
(ii) $\beta i(A) \cup \beta i(B) \subseteq \beta i(A \cup B)$.
(iii) $\beta i(A \cap B) \subseteq \beta i(A) \cap \beta i(B)$.
(iv) $\beta i\left(\beta i(A)\right) = \beta i(A)$.

Proposition 1.6.8: Let (X, τ) be a topological space and let$\{A_\propto\}_{\propto \in \Lambda}$ be a family of ssubsets of X. Then

(i) $\bigcup_{\propto \in \Lambda} \beta i\,(A_\propto) \subseteq \beta i\,(\bigcup_{\propto \in \Lambda} A_\propto)$.
(ii) $\beta i\,(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} \beta i\,\,(A_\propto)$.

Definition 1.6.3[1]: Let (X, τ) be a topological space and A ⊆ X. Then$A$ is said to be**$\boldsymbol{\beta}$ -closed set** if $X - A$ is $\boldsymbol{\beta}$ -open set in X.

Remark1.6.4: In a topological space (X, τ) the empty set $\emptyset$ and whole set X are $\beta -$ closed sets.

Proposition1.6.9: In a topological space (X, τ) each closed set is β-closed but not the converse.

Proposition1.6.10: In a topological space (X, τ) each $\alpha -$closed set is β-closed but not the converse.

Proposition1.6.11: In a topological space (X, τ) each semi- closed set is β-closed but not the converse.

Proposition1.6.12: In a topological space (X, τ) each pre- closed set is β-closed but not the converse.

Theorem1.6.3 [1]: Let (X, τ) be a topological space and let A ⊆ X. Then following are equivalent:

(i) A is $\beta -$closed**.**
(ii) $Int(Cl(Int(\mathrm{A}))) \subseteq A$.

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(iii) $Int(A)) = Int(Cl(Int(A)))$.

Proposition1.6.13 [1]: In a topological space (X, τ) arbitrary intersection of $\beta -$ closed sets is β – closed**.**

Remark 1.6.5: In a topological space (X, τ) union of two β- closed sets may not beβ– closed set in X.

Definition 1.6.4 [7]: Let (X, τ) be a topological space andlet$A \subseteq X$.Then intersection of all β -closed sets in X containing A is called β **-closure** of A. The β -closure of A is denoted by $\boldsymbol{\beta}c(A)$.

Remark 1.6.6: We note that $\boldsymbol{\beta}c(A)$ is the smallest $\boldsymbol{\beta}$-closed set in (X, τ) containing A.

Proposition 1.6.14: Let (X, τ) be a topological space and$A \subseteq X$. Then A is $\boldsymbol{\beta}$-closed set if and only if $\beta c(A) = A$.

Proposition 1.6.15: Let(X, τ) be a topological space and let A, B be subsets of *X*. Then following properties holds:

(i) $\beta c(\phi) = \phi, \beta c(X) = X$.
(ii) If $A \subseteq B$then $\beta c(A) \subseteq \beta c(B)$.
(iii) $\beta c(A) \cup \beta c(B) \subseteq \beta c(A \cup B)$.
(iv) $\beta c(A \cap B) \subseteq \beta c(A) \cap \beta c(B)$.
(v) $\beta c\left(\beta c(A)\right) = \beta c(A)$.

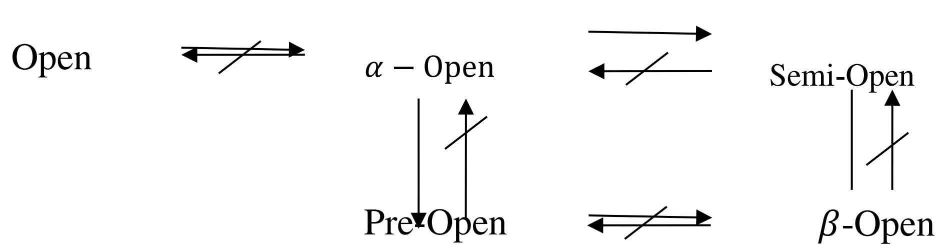
Proposition 1.6.16: Let (X, τ) be a topological space and let $\{A_\propto\}_{\propto \in \Lambda}$be a family of subsets of X. Then

(i) $\bigcup_{\propto \in \Lambda} \beta c\,(A_\propto) \subseteq \beta c\,(\bigcup_{\propto \in \Lambda} A_\propto)$.
(ii) $\beta c(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} \beta c(A_\propto)$.

Proposition 1.6.17: Let (X, τ) be a topological space and let$A \subseteq X$.Then

(i) $\beta i\,(X - A) = X - \beta c(A)$.
(ii) $\beta c(X - A) = X - \beta i\;(A)$

Remark 1.6.10: We can summaries the above results in this section in the following diagram:



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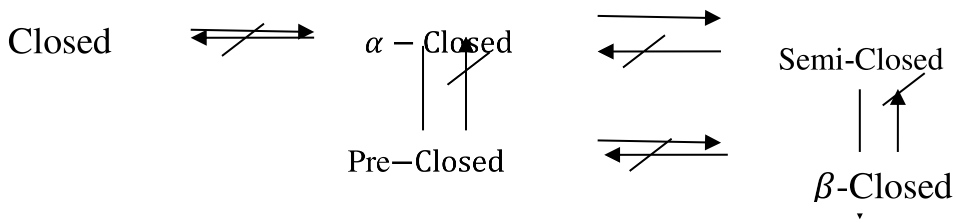

$$\text{Int(A)} \subseteq \alpha i(A) \subseteq Si(A) \subseteq \beta i(A) \subseteq A$$

$\subseteq Pi(A)$

$$A \subseteq \beta c(A) \subseteq Sc(A) \subseteq \alpha c(A) \subseteq cl(A)$$

$\beta i(A) \subseteq A$

$\supseteq Pc(A) \supseteq \beta c(A) \supseteq A$

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