IJFANS INTERNATIONAL JOURNAL OF FOOD AND NUTRITIONAL SCIENCES

ISSN PRINT 2319 1775 Online 2320 7876

Research paper UGC CARE Listed (Group -I) Journal Volume 11, Iss 13, 2022

Reviewing Advanced Numerical Methods For Solving Differential Equations

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Abstract

Numerical methods play a crucial role in approximating solutions to differential equations across various scientific and engineering disciplines where analytical solutions are elusive or impractical. This abstract explores advanced numerical techniques tailored for solving differential equations, focusing on their efficacy, accuracy, and computational efficiency.

The finite element method (FEM) stands prominent among these methods, offering robust solutions for problems ranging from structural analysis to fluid dynamics. By discretizing the domain into finite elements, FEM transforms complex partial differential equations into algebraic equations, thus facilitating numerical approximation. Its versatility extends to nonlinear problems, adaptive mesh refinement, and parallel computing, enhancing scalability and performance. Another pivotal approach is the spectral method, leveraging orthogonal basis functions like Fourier or Chebyshev polynomials to achieve high-order accuracy. Spectral methods excel in problems with smooth solutions, exhibiting rapid convergence but demanding regularity in the solution domain. the finite volume method (FVM) emphasizes conserving quantities across discrete volumes, making it ideal for fluid flow simulations and transport phenomena. Its structured grid framework ensures robustness under complex geometries and unstructured grids, catering to diverse engineering applications. This research reviews these methods' theoretical foundations, implementation challenges, and comparative advantages, emphasizing their applicability and limitations in practical scenarios. Understanding their computational intricacies and trade-offs aids researchers and practitioners in selecting the optimal numerical strategy for addressing specific differential equation problems.

Introduction

The introduction to the review of advanced numerical methods for solving differential equations provides a comprehensive overview of the evolution, significance, and current state of these methods in scientific and engineering disciplines. Differential equations serve as fundamental tools for modeling a wide array of phenomena in natural and applied sciences. While analytical solutions exist for some differential equations, many real-world problems necessitate numerical approaches due to their complexity or lack of closed-form solutions. Advanced numerical methods have thus emerged as indispensable tools for tackling these challenges. Historically, numerical methods like Euler's method laid the foundation for

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computational mathematics in the 18th century, followed by more sophisticated techniques developed throughout the 20th and 21st centuries. The advent of computers revolutionized numerical analysis, enabling the development of iterative methods such as the Runge-Kutta methods, finite difference methods, and finite element methods. These methods discretize continuous problems into manageable forms, allowing for approximate solutions through iterative computation. The significance of advanced numerical methods lies in their ability to provide insights into complex systems that cannot be fully understood through analytical means alone. They are extensively applied in fields such as fluid dynamics, structural analysis, climate modeling, and optimization problems. By simulating and predicting behaviors of intricate systems, these methods facilitate decision-making processes and the design of innovative technologies.

Current research focuses on enhancing the accuracy, stability, and efficiency of numerical methods to address ongoing challenges in computational science. Issues such as numerical stability, convergence properties, and scalability remain critical areas of investigation. Moreover, validation and verification techniques continue to evolve to ensure the reliability and robustness of numerical simulations in practical applications. the review of advanced numerical methods for solving differential equations aims to explore the evolution of these methods, highlight their significance across diverse disciplines, and discuss ongoing research efforts aimed at overcoming existing challenges. By advancing these techniques, researchers aim to further expand the boundaries of computational modeling and simulation, fostering innovation and improving our understanding of complex systems in the natural and engineered world.

Literature Review

Khan, Kamil & Ali, Arshed & Fazal-i-Haq (2021) This article examines the development of two numerical techniques for solving a partial integro-differential equation (PIDE) of the convection-diffusion type with a weakly singular kernel. Interpolation is accomplished through the use of cubic trigonometric B-spline (CTBS) functions in both approaches. The initial approach is a CTBS-based collocation method, which, once applied, transforms the PIDE into an algebraic tridiagonal system of linear equations. CTBS-based differential quadrature is the other method, and it is the method that converts the PIDE to a system of ODEs by computing spatial derivatives as a weighted sum of function values. Both the solution to the linear system that was obtained in the first method and the determination of the weighting coefficients in the second method are carried out with the assistance of an effective tridiagonal solver. For the purpose of finding a solution to the system of ODEs that was obtained through the second method, an explicit scheme was used as a time integrator. For the purposes of validating the methods, they are evaluated using three different types of heterogeneous problems. Analyses are performed to examine the stability, computational efficiency, and numerical convergence of the methods. A comparison is made between the errors in approximations produced by the current methods and the errors produced by different discretization parameter values and convection-diffusion coefficient values. Peclet

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number is used to discuss convection dominant cases as well as diffusion dominant cases. In addition to this, the results are contrasted with the cubic B-spline collocation method.

Jackson, R. (2021) The simplest equation method, often known as the SEM, is now considered to be the most reliable approach to solving [nonlinear] partial differential equations, also known as [N]PDEs. The G'/G-expansion method is an additional approach that was used in the past for the purpose of solving equations of this kind. This approach seems to derive from the simplest equation method (SEM). This research looks at a novel approach to solving partial differential equations (PDEs) known as the generating function technique (GFT), which has the potential to set a new precedent for SEM. First, the research delves into the connection between GFT and SEM as well as the G'/G-expansion approach. Following this, the authors of the study provide a novel theorem that demonstrates how solutions to PDEs may be derived using both GFT and Ring theory. The unique approach is then used in the process of deriving new or unusual solutions to equations such as the Benjamin-Ono equation, a QFT (nonlinear Klein-Gordon equation), and a Good Boussinesq-like equation. In the end, the research comes to a close with a discussion of the reasons why the approach is superior than SEM and the G'/G-expansion method, as well as the potential applications of GFT in the field of mathematics, particularly differential equations.

Butcher, John (2021) Since the topic of discussion in this book is the algebraic analysis of numerical methods, having a strong foundation in both ordinary differential equations and numerical methods for resolving them is essential before reading this book. In this chapter, a very general overview of these significant topics is provided for your perusal. There will be a discussion on the fundamental theory behind initial value problems, in part by utilising a variety of test problems. These issues are brought about by the application of conventional physical modeling, to which a number of fabricated and manufactured issues have been added. After this, a cursory examination of the traditional one-step and linear multistep methods, followed by an even cursorier examination of some all-encompassing multivalue-multistage methods (also known as "general linear methods"), is presented. Some of the procedures are accompanied by numerical examples, which highlight some of the properties that they possess. B-series, trees, and elementary differentials are all given a cursory introduction here as a sort of teaser for what will come in later chapters.

Yitayew, Tigist & Ketema, Teketel (2020) The primary objective of this study is to demonstrate how first-order ordinary differential equations may be used as a mathematical model, especially for elucidating certain biological processes and mixing issues. The application of first order ordinary differential equations in modelling some biological phenomena, such as the logistic population model and the prey-predator interaction for three species in a linear food chain system, has been investigated. This research was carried out. In addition, the use in drug mixing issues has been proved in both single tank and multiple tank systems. In conclusion, it has been shown that, when it comes to modelling a population model, the logistic model is far more powerful than the exponential model.

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Kumar, Dr. Sunil & Shaw, Pawan & Abdel-Aty (2020) In this study, we devised two numerical techniques that are both economical and quick in order to solve an initial value problem (IVP) for linear and nonlinear fractional differential equations (FDEs) of order, where 0 is less than 1 and greater than 1. In this example, we have applied the Riemann style to the arbitrary order derivatives. The suggested technique is very accurate and delivers the answers in a straightforward manner, free of any assumptions on linearization, perturbations, or any other factors. To demonstrate the efficacy and precision of our algorithm, illustrative examples and numerical comparisons are provided to contrast it with the exact method, the Euler method, and the improved Euler method (IEM), respectively. When it comes to solving the IVP of FDEs, this approach has a quicker convergence rate than the Euler method and the IEM thanks to its quadratic and cubic convergence rates. In addition, we have explained the behaviours via the use of graphical representations of the solutions that we have acquired. In addition, the use of either approach will be helpful in the treatment of illness models in further research.

Injrou, Sami & Karoum, Ramez & Hilal, Nayrouz (2019) A numerical iterative approach for calculating approximate solutions to the Newell-Whitehead-Segel partial differential equation is presented in this study. This method is dependent on replacing the derivatives of the numerical solutions with suitable approximations, which can be derived from the definition of the derivative. This results in the partial differential equation transferring to a system of nonlinear algebraic equations, which can be solved using the Newton iterative method. By solving two separate problems and comparing the results with other numerical results, as well as by studying the consistency and stability of the proposed method, the accuracy of the numerical solution and its consistency with the exact solution are tested. The accuracy of the numerical solution and its consistency with the exact solution are tested.

Lyu, Pin & Vong, Seakweng (2019) In this study, we propose and investigate an effective numerical method for solving nonlinear Caputo q-fractional differential equations with nonsmooth solutions. This method was developed by the authors of this work. The numerical method is a linearized one that can be applied to the graded mesh for implementation. The method converges to the second order on uniform mesh for equations that have fractional derivatives of order (1, 2). When the grading parameter of the nonuniform mesh is given the appropriate value, it is possible to obtain second-order convergence for that falls between 0 and 1.

Wang, Youyu & Liang, Shuilian & Wang, Qichao (2018) This article discusses a novel kind of fractional differential equation of arbitrary order that may be obtained by combining an integral boundary condition with a multi-point boundary condition. The Green's functions may be derived by first resolving the equation that represents the issue that is going to be the focus of our investigation. The existence of multiple positive solutions for the BVPs is proven based on some properties of Green's functions and under the circumstance that the continuous functions f satisfy certain hypothesis. This is accomplished by defining a continuous operator on a Banach space, making use of the cone theory, and utilising some

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fixed point theorems. In conclusion, several examples are given to demonstrate the outcomes of the study.

Hassan, Aliyu & Zakari, Yahaya (2018) which is the location where the model is constructed. After that, we apply techniques or computer-aided numerical computation in order to manipulate the model. In the end, we emerge back into the real world, carrying with us the answers to the mathematical issues, which may then be adapted into answers that are helpful to the problems that really exist. The method of separation of variables and Newton's law of cooling were used to find the solution of the temperature problems that requires the use of first order differential equation, and these solutions are very useful in mathematics, biology, and physics, particularly in analysing problems involving temperature that require the use of Newton's law of cooling. The application of first order differential equation in temperature has been studied. The method of separation of variables was used. Newton's law of cooling was used.

Rabiei, Faranak & Hamid, Fatin & Ismail, Fudziah (2017) A generic linear approach is given for use in the numerical simulation of fuzzy differential equations that is presented in this paper. The importance of deriving the algebraic order conditions of the method by using the technique of rooted trees and B-series is what gives the general linear method its significance. For the purpose of solving fuzzy differential equations, a fuzzy general linear method of order 3 that is based on the concept of generalised Hukuhara differentiability has been developed. It has been shown that the third-order fuzzy general linear technique converges. The proposed method is validated by applying it to fuzzy initial value problems. The numerical results demonstrated that the fuzzy general linear method produced a more accurate approximation of the fuzzy solution for the problems that were put to the test when compared to the existing fuzzy numerical methods.

Ahmad, Najmuddin & Charan, Shiv (2017) In the last discussion, we looked at an ordinary differential equation of the first order with a boundary condition. Using MATLAB, we were able to solve these equations using Euler's improved technique, Euler's modified method, and the Runge-Kutta fourth order approach. All three of these methods were modified by Euler. We have estimated the value of y at each point along the interval, and then compared it to the value that it really has at that time. The disparity between the calculated and precise numbers represents the amount of error in the value of y. In addition, the percentage of error has been computed for each individual point along the intervals. When the results of the Euler's improved technique, the Euler's modified method, and the Runge-Kutta fourth order approach are compared, it can be shown that the Runge-Kutta fourth order method performs much better in every scenario. The Runge-Kutta fourth order approach performs much better than Euler's enhanced method and Euler's modified method, according to the mean of the findings for all differential equations.

Sigamani, Valarmathi & Miller, John & Narasimhan (2016) This book is a great resource for academics working in the subject of differential equations since it provides an excellent introduction to singular perturbation issues and serves as a beneficial guide. In addition to this, it has chapters that discuss recent developments in both of these areas: singular

ISSN PRINT 2319 1775 Online 2320 7876

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perturbation issues and differential equations. This book serves as a comprehensive source of information on the underlying ideas in the construction of numerical methods to address different classes of problems with solutions of different behaviours. This will ultimately help researchers to design and evaluate numerical methods for the purpose of solving new problems. The book was written by experts who are active researchers in the related fields. The images in the form of tables and graphs that are offered throughout the book serve to enhance each of the chapters that are presented.

Çakir, Musa (2016) The focus of the current research is on the numerical solution to a singly perturbed semilinear boundary value problem using the finite difference technique on a piecewise uniform mesh (Shishkin type mesh). The integral boundary condition is also a part of the equations that must be satisfied. Before offering a technique for the numerical solution to the singly perturbed differential problem, we will first examine the nature of the continuous solution to the issue and then provide that approach. The piecewise uniform Shishkin type mesh is the basis for the construction of the numerical approach. We demonstrate that the approach is first-order convergent in the discrete maximum norm, with the exception of a logarithmic factor, and this holds true regardless of the singular perturbation value. The nonlinear difference issue may be effectively solved using the iterative approach that we provide. There is a presentation of numerical findings that back up the estimations that have been offered.

C. Remesan, Gopikrishnan (2015) The Generalized Auto Regressive Conditional Heteroskedasticity process (GARCH) is a discretization method that belongs to a class of methods that are extremely important due to the ease of use as well as the robustness that they possess. This is one of the many volatility estimation schemes that can be derived from data that has been collected in the past. As a result, it is essential to acquire an understanding of the behaviour of this discretized scheme when it is applied in a continuously extended fashion. The aforementioned argument is supported by the recent uptick in the number of publications coming from this subfield of quantitative finance. Even though several continuous time extensions for the GARCH process have been proposed under a variety of different assumptions and parameterizations, the delay integro differential formalism stands out as the one that appears to be the most natural. Even though deterministic and stochastic integro differential equations have received a significant amount of research, very little research has been conducted on stochastic delay integro differential equations. Establishing important properties of stochastic delay integro differential equations is the primary purpose of this thesis. [Citation needed] [Citation needed] We have made an effort to demonstrate a theorem that is analogous to the existence and uniqueness of solutions for stochastic delay integro differential equations. Other significant aspects of the solutions, such as their boundedness in Lp and their stability, are also investigated. We attempted to give approximate numerical solutions to a class of equations by employing the well-known Euler - Maruyama scheme, but it was nearly impossible to solve such equations analytically. As a result, we determined an error bound for the solutions and tried to give approximate numerical solutions of those equations. Using the tools described above, we attempted to solve a particular SDIDE in mathematical finance that models volatility. This is the part of the project where we come to a conclusion.

Chen, Yiming & Wei, Yan-Qiao & Liu, Da-Yan & Yu, Hao (2015) A numerical method for solving a class of nonlinear variable order fractional differential equations is proposed in this article. The Legendre wavelets functions and operational matrices are what are going to be used for this. In the first place, a family of piecewise functions is suggested, and based on these functions, it is simple to calculate the variable order fractional derivatives of the Legendre wavelets functions. In the second step of the process, operational matrices are derived in order to convert the FDEs that have been studied into a set of algebraic equations. After that, numerical solutions are found by working through these equations step by step. In the final part of this presentation, some numerical examples are given to illustrate how accurate the proposed method is.

Roos, Hans-Görg & Stynes, Martin (2015) This is a preprint version of an article that will appear in Computational Methods in Applied Mathematics — Several unanswered topics pertaining to the numerical analysis of singly perturbed differential equations are treated here. These include the question of whether or not certain convergence results in various norms are optimal, the question of when supercloseness is obtained in finite element solutions, the question of whether or not defect correction in finite difference approximations is valid, and the question of whether or not desirable adaptive mesh refinement results have yet to be proved or disproved.

Begam, Jannath & Rajan, Praveen Kumar (2015) In this study, we solve partial differential equations in three dimensions with variable coefficients using the Reduced Differential Transform Method (RDTM) and compare it to the Differential Transform Method (DTM). Numerical examples are provided for implementing both approaches. The findings indicate that RDTM is a straightforward method that produces extremely good outcomes.

Research Problem

The research problem in the study of advanced numerical methods for solving differential equations revolves around addressing the limitations and enhancing the capabilities of existing computational techniques. Despite significant advancements in numerical methods, several challenges persist that hinder their widespread application and efficacy across various scientific and engineering domains. One primary challenge is the trade-off between accuracy and computational efficiency. Many numerical methods require a balance between achieving high accuracy in solutions while maintaining reasonable computational costs, especially for large-scale problems. Improving upon existing methods to achieve both accuracy and efficiency remains a critical research problem. Another significant issue is the stability and convergence of numerical schemes. Some methods may exhibit instability or convergence issues when applied to certain types of differential equations, particularly those with stiff or oscillatory solutions. Developing robust numerical schemes that ensure stability across diverse problem domains is essential for reliable predictions and simulations. the scalability

ISSN PRINT 2319 1775 Online 2320 7876

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of numerical methods poses a challenge, particularly with the increasing complexity and size of modern computational models. Methods that perform well on smaller scales may struggle to scale effectively to larger systems, necessitating research into scalable algorithms and parallel computing techniques. the validation and verification of numerical results against analytical solutions or experimental data are crucial for ensuring the reliability and credibility of computational models. Establishing rigorous validation protocols and benchmarking studies is essential to address uncertainties and validate the accuracy of numerical predictions. these challenges requires interdisciplinary collaboration between mathematicians, computational scientists, and domain experts in fields such as physics, engineering, biology, and economics. By identifying and tackling these research problems, advancements in advanced numerical methods can significantly impact scientific research, engineering practice, and technological innovation, paving the way for more accurate simulations, improved designs, and better-informed decision-making processes.

Conclusion

In conclusion, the study and application of advanced numerical methods for solving differential equations represent a cornerstone of modern scientific and engineering endeavors. These methods have evolved significantly, driven by the need to model complex phenomena accurately and efficiently where analytical solutions are impractical or non-existent. we have explored various advanced techniques including finite element methods, finite difference methods, spectral methods, and others. These methods have demonstrated their effectiveness in diverse fields such as fluid dynamics, structural mechanics, electromagnetics, and more recently, in multidisciplinary applications like bioinformatics and climate modeling. The importance of these methods cannot be overstated. They enable researchers and engineers to simulate and predict behaviors of complex systems with unprecedented accuracy, aiding in the design of safer structures, more efficient processes, and innovative technologies. Moreover, they play a crucial role in decision-making processes, offering insights that guide policy-making, resource management, and strategic planning. the ongoing research in this area focuses on refining numerical algorithms to improve computational efficiency without compromising accuracy, enhancing stability in solving stiff differential equations, and advancing parallel computing techniques to handle increasingly larger-scale problems. Additionally, there is a continuous effort to validate and verify numerical results against experimental data or analytical solutions to ensure reliability and build trust in computational simulations.

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