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Gradient Group Optimizer based Corona Product with Meta Heuristic for Set-Union Knapsack Problem in Group Theory

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ABSTRACT Group theory provides the conceptual framework for solving mathematical measurement problems in various fields such as computer science, mathematics, science and statistics, thus widely used in many researches. However, while performing with binary operation between group elements, solving the Binary Optimization-based Set Union Knapsack Problem (BO-SUKP) is more challenging in group theory since, the axioms (binary, associative, identity, inverse, and commutativity) of groups are not satisfied, as the problem involves selecting a subset of items from multiple sets to maximize a certain objective function while satisfying certain constraints. Hence, a novel Gradient Group Optimizer based Corona Product with Meta-Heuristic for Set-Union Knapsack Problem in Group Theory is introduced to tackle the aforementioned issues. In this novel approach, the Gradient group optimizer includes union sets to resolve the Binary Optimization (BO) variations in group theory and then Corona Product with Metaheuristic approach is developed to solve the Set Union Knapsack Problem (SUKP) by splitting up the union set's components. This proposed approach effectively solves the binary optimization issues in the set union knapsack problem over group theory by precisely satisfying the group axioms. Furthermore, the proposed model showed that the solutions are effective with best class, worst class, mean, and standard deviation.

Keywords: *Group theory, Rough set, Combinatorial problems, Optimization, Set Union Knapsack problem, and Binary optimization problems.*

1.Introduction

Group theory is a branch of pure mathematics that studies the group algebraic structure. Group theory plays a pivotal role in various fields of mathematics, science, and even in practical applications. It is a mathematical discipline that investigates the fundamental concepts of symmetry, transformation, and structure. Group Theory provides a powerful framework for understanding and classifying the symmetrical properties of objects, equations, and mathematical structures, making it an indispensable tool in diverse domains, ranging from pure mathematics to physics, chemistry, cryptography, and computer science [1-2]. At its essence, Group Theory explores the concept of a "group (G)", which is a set equipped with a binary operation (a combination rule) that satisfies specific axioms. These axioms include closure $(a + b \in G, \forall a, b \in G)$, associativity $(a + (b + c) = (a + b) + c, \forall\, a, b, c \in G)$, the existence of an identity element $(a + I = a = I + a, \forall\, a, I \in G)$, and the existence of inverses $(a + (-a) = I = (-a) + a, \forall\, a, -a, I \in G)$ for groups under addition.

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Furthermore, these four axioms are satisfied for group under multiplication also. Groups can be finite or infinite, and they can vary widely in complexity [3].

Group Theory in Physics is considered the cornerstone of modern physics, especially in quantum mechanics and particle physics. It helps describe the symmetries of physical systems, leading to fundamental insights about the behavior of matter and energy. In chemistry, symmetry and group theory are integrated to understanding molecular structures, spectroscopy, and chemical reactions [4]. They simplify complex problems by revealing patterns and regularities. In Cryptography, Cryptographic algorithms often rely on group theory principles to ensure secure data transmission and encryption. In Geometry, Euclidean geometry, which studies the properties of shapes and spaces under transformations, is deeply connected to group theory [5]. In Computer science, groups play a major role in algorithms for computer graphics, robotics, and artificial intelligence. In number theory, algebraic structures like finite fields and elliptic curves have group-theoretic properties, which are essential in cryptography and coding theory. Finally, in music and art, Group Theory is used to understand symmetries and patterns in music composition and visual art [6-7].

The Set-Union Knapsack Problem (SUKP) is a variant of the classic Knapsack Problem, a well-known optimization problem in computer science and mathematics. In the standard Knapsack Problem, given a set of items, each with a weight and a value selects a subset of these items to maximize the total value while staying within a given weight capacity [8]. The SUKP extends this concept by introducing the idea of sets of items. In SUKP, the goal is to find the optimal selection of items from a given set to maximize the total value while adhering to a constraint on the total weight. However, SUKP introduces an additional complexity by allowing items to be grouped into sets or unions, and only one item from each set can be chosen. This constraint makes the problem more challenging as it requires not only selecting individual items but also deciding which sets to include in the knapsack [9-10]. Furthermore, solving the SUKP can be more complex than solving the standard Knapsack Problem because it involves both selecting sets and items within those sets. Various optimization algorithms, such as dynamic programming or integer programming, were applied to find the optimal solution or approximate solutions depending on the problem's size and constraints. The SUKP finds applications in various real-world scenarios, such as resource allocation, portfolio optimization, and project selection, where items are categorized into groups or sets, and choosing a representative item from each set is crucial to the decision-making process. Solving SUKP typically involves dynamic programming or other optimization techniques to efficiently determine the optimal combination of sets and items for maximum value within the given weight constraint [11-13].

In recent years, Information technology has grown exponentially, which has led to numerous binary optimization issues. The two primary subfields of these issues are stochastic and deterministic optimization issues. The latter consists of continuous optimization that is neither confined nor unrestricted and discrete optimization, which is further divided into integer programming and combinatorial optimization. Combinatorial optimization deals with issues where the set of viable solutions is discrete or may be reduced to a discrete set is the main topic of the manuscript. These optimization issues are focused for this research work, because of their significant influence on real-world issues [14-15]. There are numerous approaches to resolving combinatorial optimization issues, and they are divided into exact and approximation procedures. Branch & cut and branch & bound are included in the first. In the latter, heuristic and metaheuristic methods are used. The latter have greatly surpassed exact approaches in popularity since their simplicity and findings' robustness of the combinatorial problem is of high dimension [16].

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In the existing models, every algorithm has flaws in addressing every issue in combinatorial optimization hence these researchers create new metaheuristics every year to rectify the issues in combinatorial optimization. The development of metaheuristics including Particle Swarm Optimization, Ant Colony Optimization, Artificial Bee Colony, Bat Algorithm, Cuckoo Search, Firefly Algorithm, Grey Wolf Optimizer, Sine Cosine Algorithm, Whale Optimization Algorithm, and Dragonfly Algorithm, among others, was sparked by this intense interest. One thing all of these metaheuristics have in common is that they were created to address issues with a continuous domain of variables [17]. However, these optimization algorithms do not well perform in solving set union knapsack problems. In light of the fact that the entire problem under consideration may have arbitrarily high worst-case complexity or even be intractable, it considers partial algorithms that may only halt on the generic set. Hence there is a need to provide a novel solution for solving binary optimization-based set union knapsack problem and also there is a need to determine the worst-case complexity of set union knapsack problem-solving algorithms. The main contribution of this paper are as follows:

- Problem definition and theorems for various binary optimization-based set union knapsack problems have been provided.
- To solve BO based set union knapsack problem, a novel Gradient Group Optimizer-based Corona Product with Meta Heuristic for Set-Union Knapsack Problem in Group Theory has been presented that selects suitable representation of the elements, sets, and the union operation in terms of group theory operations without binary problem by splitting the elements in the group.

2. Narrator Review

Lin et al [18] developed a hybrid binary particle swarm optimization (HBPSO/TS) to solve the SUKP. First, the quality of the search's solutions is assessed using an adaptive penalty function. This approach aims to investigate the edge of the possible solution space. On the basis of the SUKP's features, a novel position updating mechanism is then developed. The recently developed solutions have good structures that match those of previously discovered solutions. A tabu-based mutation technique is then implemented to guide the search towards new, promising locations. Finally, in order to increase the ability to exploit, a tabu search technique is constructed. To shorten the solution time, a gain updating approach is also used. However, solving the SUKP is computationally intensive, especially for large instances. In the future, PSO-TS may require a significant amount of time to converge to a solution, making it less suitable for real-time or time-critical applications.

Wei et al [19] introduced a multistart solution-based tabu search method to address the set union knapsack problem. To effectively examine potential solutions, the proposed algorithm combines a multistart strategy with a solution-based tabu search procedure. The proposed MSBTS method finds high-quality local optima using its solution-based tabu search procedure and avoids deep local optima traps using its multistart mechanism. The absence of parameters and the ease of design and implementation are two factors that make MSBTS appealing. However, it takes long time to find a reasonably good solution.

Feng et al [20] presented an enhanced moth search algorithm (EMS) to solve SUKP, which replaces Lévy flight with an improved interaction operator (EIO) by integrating differential mutation into the global harmony search. To promote information sharing and population diversity, EMS particularly created an upgraded interaction operator by including differential mutation into the global harmony search approach. Additionally, EMS is a useful

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alternative approach for resolving SUKP issues, but the accuracy of some instances still needs to be improved, and EMS' performance does not clearly provide proper solutions.

Wei et al [21] proposed the kernel-based tabu search algorithm, which brings together the idea of a kernel and the potent tabu search technique. The Set-union Knapsack Problem (SUKP) is a useful framework for intelligent systems and decision-making. Heuristic algorithms are helpful to identify high-quality solutions in a reasonable amount of time because of its inherent difficulty (NP-hard). The proposed approach is more practical for usage in practise because it only needs three parameters. Given that SUKP has a variety of intriguing applications, the proposed approach provides a useful resource for resolving these issues in the real world. Such applications are undoubtedly made easier by our algorithm's open source nature and high computational efficiency. However, this approach is difficult to find good solutions within a reasonable time frame for large instances.

Ozsoydan et al [22] introduced a reinforcement learning method for binary optimization issues called Q-learning. The created algorithm functions as a reinforcement and recommendation system that rates the existing algorithms, gives them prizes, and either promotes or demotes them. As a result, it calls more promising optimizers more often. Particle Swarm Optimization (PSO), Genetic method, and a combination of these algorithms called genetic-based PSO (gbPSO) are used as optimizers in the proposed Q-learning method. Therefore, adopting a variety of optimizers and accumulating more statistical data is intended to avoid local optima. Secondarily, by implementing an initial solution generation method and a triggered random immigration mechanism to maintain swarm diversity, all optimizers are further improved. A mutation procedure that reduces diversity is added on top of these treatments. However, exploring binary action in SUKP is difficult.

García et al [23] proposed a hybrid k-means cuckoo search method to solve the set union knapsack problem. In order to binarize the results produced by the cuckoo search algorithm, this hybrid binarization method uses the k-means technique. The technique was strengthened with a greedy initialization approach and a local search operator in order to make it more effective. On a medium and large scale, the set-union knapsack issue was solved using the suggested hybrid technique. However, this approach is difficult handle large-scale instances of the Set-Union Knapsack Problem

From the analysis, [18] solving the SUKP is computationally intensive, [19] takes long time to find a reasonably good solution, [20] it does not maintain a diverse population of solutions during the optimization process, [21] find it difficult to resolve knapsack problems in a timely manner, [22] does not explore binary action in SUKP and [23] it is challenging to address the SUKP in large scale instances. Hence there is a need to develop an effective approach for solving the binary optimization based set union knapsack problem in group theory.

3. Gradient Group Optimizer based Corona Product with Meta Heuristic for Set-Union Knapsack Problem in Group Theory

The set union knapsack problem, which is based on binary optimization, has been defined and discussed in this section. It also effectively investigates the effects of binary optimization in group theory by providing theoretical justifications and taking optimization variants and generic case complexity of the problems into consideration. A unique Gradient Group Optimizer based Corona Product with Meta Heuristic for Set-Union Knapsack Problem in Group Theory has been provided as a solution method for solving BO based SUKP since the effect of the binary optimization based set union knapsack issue has been theoretically

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demonstrated. This innovative solution technique operates in two distinct phases. First, it solves the knapsack problem and its optimization variations without binary complexity by adding union sets and lowering the population size in optimization. The set union knapsack problem is addressed in the second phase by dividing the elements of the union set, and the viability of the proposed approach was demonstrated by calculating the corona product in group theory.

3.1. Problem statement

This subsection defines binary optimization issues, such as set union knapsack problems, and lists their restrictions. Additionally, the theoretical justifications for the binary optimization-based set union knapsack problem have been established. The problem statement begins by defining the major decision-making difficulties.

Definition 3.1.1: Binary Optimization based Set union Knapsack problem (SUKP)

In the binary optimization where the decision variables considered only take on binary values, typically 0 or 1 in group theory. The 0-1 Knapsack Problem is commonly extended, and SUKP is obtained by adding some more factors to each item. The formulated SUKP is denoted in equation (1) and (2).

$$maximize\ A(x) = \sum_{j\in x} A_j \tag{1}$$

$$G(x) = \sum_{\substack{a\in U\\ j\in x}} g_a \leq t, \quad x \subseteq \mathfrak{U} \tag{2}$$

where g_a, A_j ,and x stand for the a^{th} element's weight in the union set of the chosen items for the jth item's profit in the subset x, and knapsack capacity, respectively.

Definition 3.1.2: Adaptive penalty function

The SUKP is a constrained binary programming problem. The viability of potential solutions is preserved throughout the tabu search. One of the most common penalty functions for maintaining feasibility during a search is the precise penalty function. Let $\alpha(x)$ represent the total number of constraints that have been violated in equation (3).

$$\alpha(x) = \max\left\{\sum_{i\in(x)} a_i - A, O\right\} \tag{3}$$

The precise penalty function can therefore be defined in equation (4):

$$\gamma(x,\varphi) = g(x) - \varphi \times \alpha(x) \tag{4}$$

where $\varphi > 0$ is a penalty parameter. The fact that the parameter φ depends on the situation at hand and that figuring out its φ value is well known. The flexible penalty function for handling the limitations by using equation (5):

$$f(X,Y) = \begin{cases} g(x) & if\ \alpha(x) = 0 \\ K - Y \times \alpha(x) & if \alpha(x) > 0\ \&\ g(x) \geq K \\ g(x) - Y \times \alpha(x) & if\ \alpha(x) \leq 0\ \& g(x) < K \end{cases} \tag{5}$$

In which K is the lesser limit of the overall greatest value of the SUKP, and $Y > 0$ is a penalty parameter. To update the K value, it is important to use the current best function value among the practical solutions. Take into account the subsequent unrestricted binary optimization problem is given in equation (6);

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$$(UP)\begin{cases} max & f(X,Y) \\ s.t & X \in \{0,1\}^n \end{cases} \quad (6)$$

3.2. Theoretical proofs

The theoretical proofs for Binary optimization problem including set union knapsack problem has been provided in this subsection.

Theorem 3.2.1: A one-dimensional representation of a group G exists in any set union knapsack problem

Proof: Take $g \in G$ and invoke the representation. Afterward, g commutes with each h. Schur's lemma and subgroup both prove that g is a scalar. Every line in U is invariant since each group element acts by a scalar, and subgroup requires that U is a line. For cyclic groups C_{n_k} of orders n_k, the structure theorem states that every finite abelian G splits as $\prod_k C_{n_k}$. Select a generator for every factor, g_k, that must act on any line via a root of unity of order n_k. Selecting such a root of unity for each k is necessary in order to describe a one-dimensional representation. A finite abelian group's order is specifically matched by the number of subgroup isomorphism classes of its representations. However, one must select generators in each cyclic component for that; there is no canonical link between group elements and representations. This claim is absolutely incorrect over various ground fields, as one could anticipate given the failure of Schur's lemma. For instance, the two $\mathbb{Z}/3$ knapsack problems over the reals have dimensions 1 and 2. They are the trivial and rotation representations. The irreps over their endomorphism ring, which is an extension field of n, can nevertheless, be demonstrated to be one-dimensional in general. Hence, a 1-dimensional representation of abelian group exists in any knapsack problem.

Remarks: The aforementioned equality applies to non-abelian finite groups, however not in the way that one may initially suppose. The number of subgroup isomorphism classes equals the number of conjugacy classes in the group is one accurate assertion. Another generalization states that the order of the group is equal to the sum of the squared dimensions of the subgroup isomorphism classes.

Theorem 3.2.2: The finite-dimensional group over n is called $End^G(U)$ if U is an subgroup finite-dimensional G-representation over n.

Proof: Let U be the subgroup finite dimensional G-representation over n. Then, ker ψ and Im ψ are G-invariant subspaces for any $\psi: U \longrightarrow U$ commuting with G.

Suppose ker $\psi = \{0\}$, then it is injective and an isomorphism (for dimensional reasons or because Im $\psi = U$, or if ker $\psi = U$, then $\psi = 0$, subgroup entails that either ker $\psi = \{0\}$ or ker $\psi = U$ and $\psi = 0$. Therefore, $End^G\ (U)$ is an finite dimensional division algebra over n.

Theorem 3.2.3: The orbit basis of G in $End(M_n^{\otimes k})$ is a subgroup of S_n.

Proof: Let $\{v_j \mid 1 \le j \le n\}$ serve as the foundation for the permutation module M_n of S_n and assume that $n, x \in \mathbb{Z} \ge 1$. The components $v_r = v_{r_1} \otimes \cdots \otimes v_{r_k}$ for $r = (r_1, \ldots, r_k) \in [1,n]^k = \{1,2,\ldots,n\}^k$ with S_n operating diagonally, k provide the basis for the S_n module $M_n^{\otimes k}$, where $\sigma.v_r = v_{\sigma(r)} := v_{\sigma(r_1)} \otimes \cdots \otimes v_{\sigma(r_n)}$. Assume that $\varphi = \sum_{r,s\in[1,n]^k} \varphi_r^s E_r^s \in End(M_n^{\otimes k})$ Where $\{E_r^s\}$ is a basis for $End(M_n^{\otimes k})$ of matrix units, and the coefficients φ_r^s are associated with F. Next, with $\delta_{r,t}$ the Kronecker delta is represented by the expression $E_r^s v_t = \delta_{r,t} v_s$, and the action of v on the basis of simple tensors is given by equation (7);

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$$\varphi(v_r) = \sum_{s\in[1,n]^k} \phi_r^s v_s \quad (7)$$

For any subgroup $G \subseteq S_n$ (in particular, for S_n itself) and for the centralizer algebra $End(M_n^{\otimes k}) = \{\varphi \in End(M_n^{\otimes k}) \mid \varphi\sigma = \sigma\varphi \; for \; all \; \sigma \in G\}$.

Therefore, $\varphi \in End(M_n^{\otimes k}) \Leftrightarrow \varphi\sigma = \sigma\varphi \; for \; all \; \sigma \in G$

$\Leftrightarrow \sum_{s\in[1,n]^k} \varphi_r^s \, v_{\sigma(s)} = \sum_{s\in[1,n]^k} \varphi_{\sigma(r)}^s v_s \; for \; all \; r \in [1,n]^k$ And so

$$\varphi \in End(M_n^{\otimes k}) \Leftrightarrow \varphi_r^s = \varphi_{\sigma(r)}^{\sigma(s)} \; for \; all \; r,s \in [1,n]^k, \sigma \in G. \quad (8)$$

It is convenient to view the pair of k-tuples $r,s \in [1,n]^k$ in equation (7) as a single 2k-tuple $(r,s) \in [1,n]^{2k}$. In this notation, condition (8) tells that the elements of $End(M_n^{\otimes k})$ are in one-to-one correspondence with the G-orbits on $[1,n]^{2k}$, where $\sigma \in G$ acts on $(r_1,\ldots,r_{2k}) \in [1,n]^{2k}$ by $\sigma(r_1,\ldots,r_{2k}) = (\sigma(r_1),\ldots,\sigma(r_{2k}))$. Then adopt the shorthand notation $(r|r') = (r_1,\ldots,r_{2k}) \in [1,n]^{2k}$ when $r = (r_1,\ldots,r_{2k}) \in [1,n]^k$ and $r' = (r_{k+1},\ldots,r_{2k}) \in [1,n]^k$. Let the G-orbit of $(r|r') \in [1,n]^{2k}$ be denoted by $G(r|r') = \{\sigma(r|r') | \, \sigma \in G\}$ and define

$$X_{(r|r')} = \sum_{(S|S')\in G(r|r')} E_S^{S'} \quad (9)$$

From equation (9), it is obtained that the sum is over the distinct elements in the orbit. This is the indicator function of the orbit $G(r|r')$, and it satisfies equation (8), so $X_{(r|r')} \in End_G(M_n^{\otimes k})$. Let $[1,n]^{2K}/G$ be the set consisting of one 2k tuple $(r|r')$ for each G-orbit. Since equation 3.4 is a necessary and sufficient condition for a transformation to belong to $End_G(M_n^{\otimes k})$, and the elements $X_{(r|r')}$ for $(r|r') \in [1,n]^{2K}/G$ are linearly independent. Hence the orbit basis of G in endomorphism is the subgroup of S_n.

Theorem 3.2.4: The subspace $V_s = \mathbb{C}[S_n]\mathfrak{c}_s$ of $\mathbb{C}[S_n]$ is an knapsack problem of S_n under left multiplication. Every knapsack problem of S_n is isomorphic to V_s for a unique s.

Proof: Let assume that $x \in \mathbb{C}[S_n]$. Then $a_s x b_s = \ell_s(x) c_s$, where ℓ_s is a linear function.

If $g \in P_s Q_s$, then g has a unique representation as $pq, p \in P_s, q \in Q_s$, so $a_s \, g \, b_s = (-1)^q c_s$. Thus, to prove the required statement, first show that if g is a permutation which is not in $P_s Q_s$ then $a_s \, g \, b_s = 0$.

To show this, it is sufficient to find a transposition t such that $t \in P_s \; and \; g^{-1} t g \in Q_s$; then the condition is given in equation (10);

$$a_s \, g \, b_s = a_s \, t_g \, b_s = a_s g(g^{-1} t g) b_s = -a_s \, g \, b_s. \quad (10)$$

So that $a_s \, g \, b_s = 0$. In other words, to find two elements i, j standing in the same row in the tableau $T = T_s$, and in the same column in the tableau $T' = gT$ (where gT is the tableau of the same shape as T obtained by permuting the entries of T by the permutation g). Thus, it suffices to show that if such a pair does not exist, then $g \in P_s Q_s$, i.e., there exists $p \in P_s, q' \in Q_s' := g Q_s g^{-1}$ such that $pT = q'T'$ (so that $g = pq^{-1}, q = g^{-1} q' g \in Q_s$). Any two elements in the first row of T must be in different columns of T', so there exists $q_1' \in Q_s'$ which moves all these elements to the first row. So there is $p_1 \in P_s$ such that $p_1 T$ and $q_1' T'$ have the same first row. Now do the same procedure with the second row, finding elements p_2, $q_2{}'$ such that $p_2 p_1 T$ and $q_2{}' q_1{}' T'$ have the same first two rows. Continuing so, it will construct the desired elements p,q'.

3.3 Gradient Group Optimizer based corona product with Meta Heuristic

A novel Gradient Group Optimizer based corona product with Meta Heuristic has been presented as a solution strategy for addressing the effects of specific restrictions and constraints after analysis of binary optimization issues like the set union knapsack problem and their definitions, as well as an effective investigation of the impact of binary optimization problems in group theory by providing the theoretical proofs along with considering the binary optimization variants and constraints, has been proven theoretically. This creative problem-solving approach operates in two stages. It addresses group theory difficulties in the first phase by offering solutions to binary optimization problems. The set union Knapsack problem is solved using group and gradient optimizers in the second phase to address the more challenging aspects of the problem. The binary optimization problem is also used to demonstrate the success of the proposed strategy.

3.3.1 Gradient Group Optimization based corona product

A set of decision variables, restrictions, and an objective function are all components of an binary optimization problem. A parameter for switching from exploration to exploitation (φ) and a probability rate are two of the GGO's control parameters. Depending on the complexity of the task, the population size and the number of iterations is considered for set union Knapsack problem. Each person in the population is referred to as a "vector" in the proposed method. V vectors are thus included in the GGO's N-dimensional search space. Consequently, a vector is mentioned in equation (11)

$$X_{m,n} = [X_{m,1}, X_{m,2}, \dots \dots ., X_{m,N}];\ \ m = 1,2, \dots, M;\ \ n = 1,2, \dots \dots ., N \qquad (11)$$

Usually, the GBO's initial vectors are created at random in the D-dimensional search domain, which is specified as in equation (12):

$$X_n = X_{min} + rand\ (0,1) \times (X_{max} - X_{min}) \qquad (12)$$

Where X_{min} and X_{max} are decision variables for bounded sets of X_n and $rand$ (0,1) is a random number in [0,1]. The gradient group rule directs the movement of the vectors to improve search in the feasible domain and obtain better placements. Based on the idea of the gradient group technique, the Gradient group optimizer is suggested with the intention of boosting the exploration tendency and speeding up the convergence of the GGO. The use of a numerical gradient group approach in place of the function's direct derivation is made due to the fact that many optimization problems are not differentiable. In general, the GGB approach starts with a guessed initial solution and advances to the next place along a gradient group-specified direction..The gradient group Optimization algorithm solved the binary optimization based SUKP in group theory.

Let $H = \{H_0, V_1\}, \dots \dots \dots . . (H_t, V_t)\}$ be a set of instances of the constraints. So, $V(H_p) = [n]$ and $|Vp| = K$, for every $P \in [t]$. Moreover, assume for each $V_i{}^P, V_j{}^p \in V_p$, $|E\ (V_i{}^P, V_j{}^P)| = M$ and $|V_i| = |V_j| = N$, for every $P \in [t]$, where $E\ (V_i{}^P, V_j{}^P)$ denotes the set of vectors between $V_i{}^P$ and $V_j{}^P$. Assume that $|H|$ is a power of two, if it's not an odd almost t copies of any instance H without changing the outcome of the composition. It denoted by (G, L, h) the instance of binary optimization, where G is the input variable, and L

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is the Q alignment. h: $[q] \rightarrow N$ is the function describing the number of targets of each issue and the goal is to find the set union problems of G so that $|\varphi_i| = h(i)$ for every iϵ $U_{v\epsilon V(G)}L(V)$. For the remainder of the solution, q is the number of constrains. The interpretation of each vector UVϵE (H, P) as the pair of sub vectors (u,v) and (v, u) and a set of 2 binary vectors. If the vector belongs to an instance (H_p, V_p) contains the solution and (u, v) is the part of (H_p, V_p).

3.3.2 Enhanced Meta Heuristic Method

The enhanced meta-heuristic method has solved the SUKP in the group theory by utilizing corona products in graphs. The partition of vertices among the different problems in knapsack is common as possible. To obtain each issue in optimizer is equitable, but typically use many optimal equitable for corona product as represented in equation (1),

$$X_{(N_{1+}N_{2+M})\times L} = \begin{pmatrix} \pi_a \\ \pi_b \\ \pi_c \end{pmatrix} \tag{13}$$

From equation (13); $\pi_a = (\pi^L{}_{a_i})$, $\pi_b = (\pi^L{}_{b_j})$, $\pi_c = (\pi^L{}_{c_m})$ Here, $\pi^L{}_{a_i}$, $\pi^L{}_{b_j}$, $\pi^L{}_{c_m}$ represents the optimized errors by at most one respectively.

$$P_{(N_{1+}N_{2+M})\times L} = \begin{pmatrix} T_a \\ T_b \\ T_c \end{pmatrix} \tag{14}$$

From equation (14), $T_a = [P^L{}_{a_i}]$, $T_b = [P^L{}_{b_j}]$, $T_c = [P^L{}_{C_m}]$ where $P^L{}_a, P^L{}_{b_j}$, $P^L{}_{c_m}$ denotes the partition of the vertices into classes which is exactly two vertices per class.

$$\left.\begin{aligned} \gamma^L{}_{a_i} &= \frac{\pi^L{}_{a_i} P^L{}_{a_i} g^L{}_{a_i}}{\sigma^2 + \sum_{m\epsilon M} \pi^L{}_{c_m} I^L{}_{c_m} a_i + \sum_{j\epsilon M_2} \pi^L{}_{b_j} I^L{}_{b_{j}a_i}} \\ \gamma^L{}_{B_j} &= \frac{\pi^L{}_{b_i} P^L{}_{b_n} g^L{}_{b_i}}{\sigma^2 + \sum_{i\epsilon M_1} \pi^L{}_{a_i} I^L{}_{a_i b_j} + \sum_{n\epsilon N} \pi^L{}_{c_n} I^L{}_{c_n b_j}} \\ \gamma^L{}_{c_m} &= \frac{\pi^L{}_{c_m} P^L{}_{c_m} g^L{}_{c_m}}{\sigma^2 + \sum_{i\epsilon M_1} \pi^L{}_{a_i} I^L{}_{a_i b_j} + \sum_{m\epsilon M} \pi^L{}_{c_{m'}} I^L{}_{c_{m'} D_m}} \end{aligned}\right\} \tag{15}$$

From equation (15), $a_i$and b_j is obtained in equation (16),

$$X^L a_i = \log_2(1 + \gamma^L a_i) \tag{16}$$

$$X^L b_j = \log_2(1 + \gamma^L c_m) \tag{17}$$

Therefore, solving the SUKP with corona product is expressed as in equation (18),

$$T = \sum_{L=1}^{m} \frac{M}{L} \left[\sum_{i=1}^{N_1} X^L a_i + \sum_{j=1}^{N_2} X^L b_j + \sum_{m=1}^{j=1} X^L c_m\right] \tag{18}$$

As a result, it follows that the measure of the set of elements that can be computed by M is zero. Overall, it is found that the algorithm's complexity is

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$X^L a_i = \log_2(1 + \gamma^L a_i)$ and with some logarithmic factors omitted if assume that $\gamma > 0$ exists and that $T = \gamma$ is true. Hence metaheuristic approach solves the set union knapsack problem and their binary optimization variants are eliminated.

Overall, theoretical proof has been provided to define the binary optimization-based set union knapsack problems. Then, by lowering the constraints, a novel method has been proposed to address set union knapsack issues with binary operators. This method resolves both binary problems and optimization variants and corona product calculations have demonstrated the usefulness of the proposed approach. The results for the Set-Union Knapsack Problem in Group Theory from the Gradient Group Optimizer based Corona Product with Meta Heuristic are detailed in the next section.

4. Results

This section contains results from the proposed methodology. The results demonstrated that the proposed approach successfully resolved the binary optimization-based Set Union Knapsack Problem, and the usefulness of the proposed strategy is also demonstrated by compared it with other current approaches including ASUKP, GA, BABC, Bin DE, and GPSO [24].

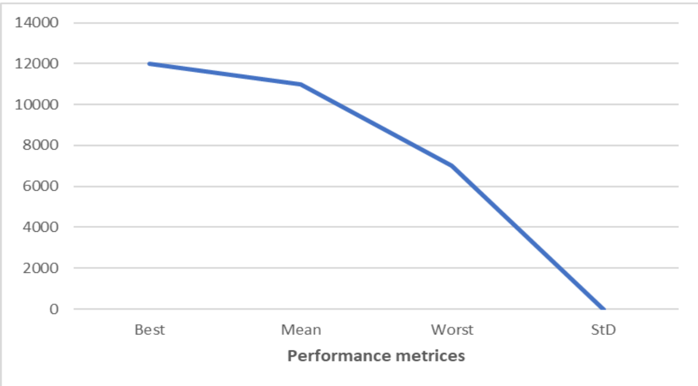

Figure 1: Performance metrics of proposed approach

Figure 1 depicts the mean, standard deviation, best class, and worst class performance metrics from the proposed approach. The ideal values for the best class prediction of elements, the worst class prediction of elements, and the mean of the proposed method are 12000, 11500, and 7000, respectively. Gradient Group Optimization, which efficiently solves binary optimization issues to find the best, worst, and mean values of group elements while also reducing the element dimensions, provides these optimum values. The Corona Product approach, which minimises binary optimization complexity by dividing the elements into sets, reduces the standard deviation of the proposed approach to 150.6. The effectiveness of this approach is confirmed by an improved meta heuristic approach, which demonstrates that the proposed approach has a low standard deviation of 150.6. The results from the proposed method have been compared with those from other approaches including ASUKP, GA, BABC, Bin DE, and GPSO in terms of the best class, the worst class, the mean, and the standard deviation.

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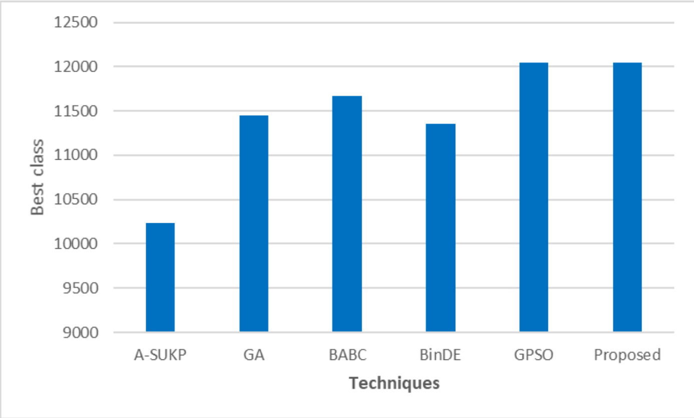

Figure 2: Comparison of Best class for the proposed approach

The best class of the proposed strategy is compared to other approaches in Figure 2. The best class of the proposed method is compared with established methods as ASUKP, GA, BABC, Bin DE, and GPSO. The best class of the proposed method has an optimal value of 12050, while the best classes of ASUKP, GA, BABC, Bin DE, and GPSO are 10231, 11454, 11664, 11352, and 12045, respectively. The best class of the proposed technique is high, whereas the best class of the ASUKP is low.

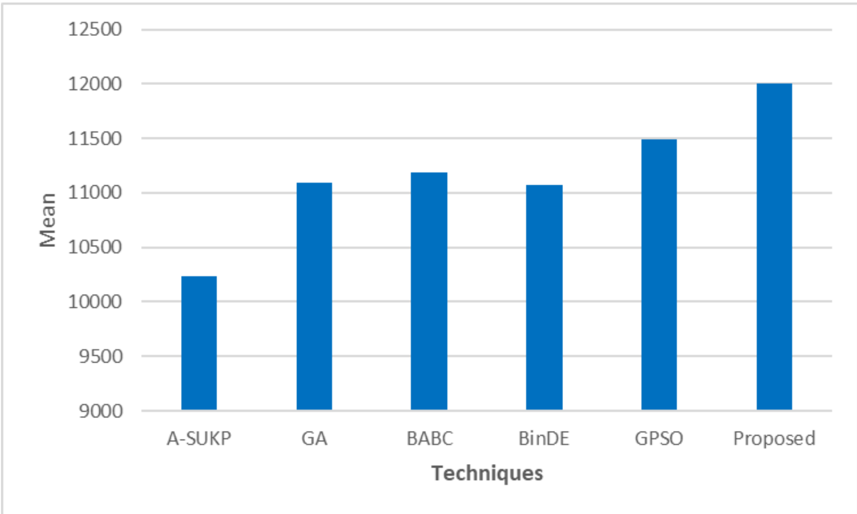

Figure 3: Mean comparison of proposed approach

Figure 3 depicts the comparison of mean of proposed approach with other existing approaches. The proposed approach's mean is compared with already-in-use methods like ASUKP, GA, BABC, BinDE, and GPSO. The mean of the proposed approach has the best value of 12000, while the means of the ASUKP, GA, BABC, BinDE, and GPSO are 10231, 11092.7, 11182.7, 11075, and 11486.95 respectively. While the mean of ASUKP is low, the mean of the proposed technique is high.

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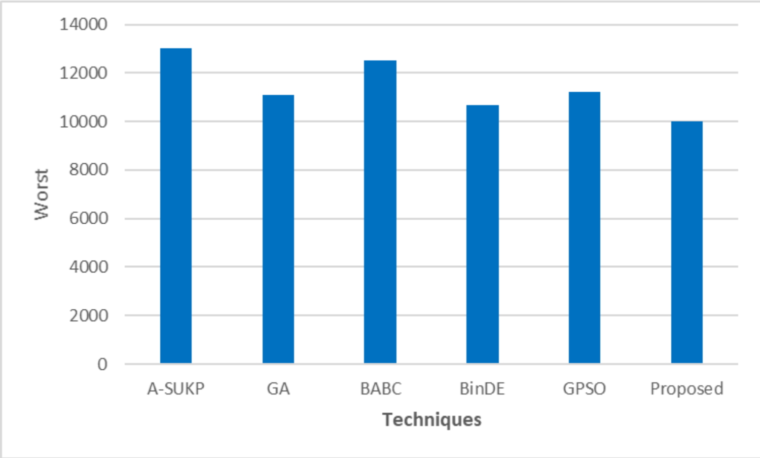

Figure 4: Worst class comparison of proposed approach

Figure 4 compares the worst class of the proposed approach with various existing approaches. The worst class of the proposed method is compared with recent approaches such as ASUKP, GA, BABC, BinDE, and GPSO. The worst class of the proposed approach has an optimal value of 10,000 while the worst classes of ASUKP, GA, BABC, BinDE, and GPSO have respective values of 13000,111092.7,12500,10675 and 11202. The proposed approach's worst class is low, whereas the ASUKP worst class is high.

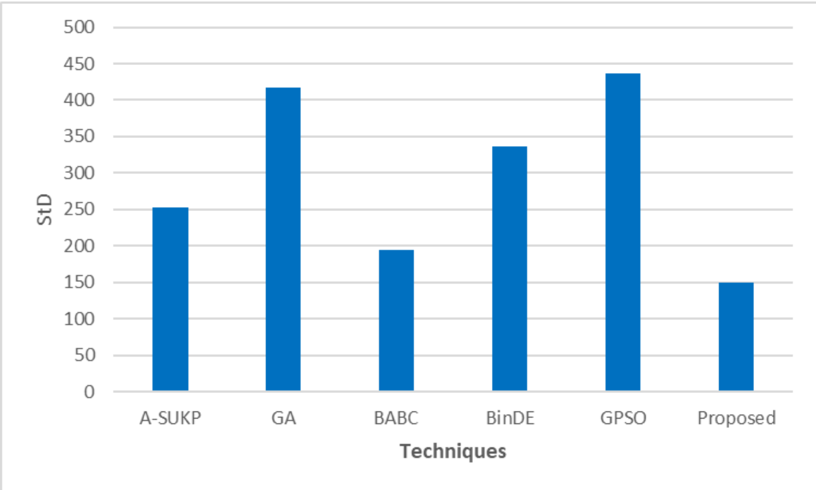

Figure 5: Standard deviation comparison of proposed approach

The standard deviation of the proposed approach is compared to other existing approaches in figure 5. The standard deviation of the proposed strategy is contrasted with existing models such as ASUKP, GA, BABC, BinDE, and GPSO. The standard deviation for the proposed approach is 150.6, while the standard deviations for the ASUKP, GA, BABC, BinDE, and GPSO are 252, 417, 193.79, 336.94, and 436.81 respectively. Here the Standard deviation of the proposed method is minimum whereas the existing GPSO has maximum optimal value.

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Table 1: Comparison of proposed approach with existing techniques

Techniques	Best	Mean	Worst	StD
ASUKP	10231	10231	13000	252
GA	11454	11092.7	11092.7	417
BABC	11664	11182.7	12500	193.79
BinDE	11352	11075	10675	336.94
GPSO	12045	11486.95	11202	436.81
Proposed	12050	12000	10000	150.6

Table 1 compares the proposed method to existing techniques such as ASUKP, GA, BABC, BinDE, and GPSO in terms of best class, worst class, mean, and standard deviation. From table 1, it is noted that the proposed approach has optimum best class, mean, and worst class values in addition to having a low standard deviation. Overall, Gradient Group Optimization based corona products for Binary optimization problems in Group theory outperform current approaches such as ASUKP, GA, BABC, BinDE, and GPSO with optimal best class, worst class, and mean values of 12050, 10000, and 12000 and have low standard deviation of 150 by metaheuristic approach.

5. Conclusion

Gradient Group Optimization based corona product with meta heuristic for binary optimization problems in Group theory has been presented in this research to solve the BO issues particularly in set union knapsack problem by performing two phases of operation. The set union knapsack problem and its optimization variants are solved in the first phase by gradient group optimization, which arranges the group's elements in ascending order according to the objective function and reduces the number of group members in order to achieve a high-quality solution with optimal best-case elements of 12050 and worst-case elements of 10000. The elements are then divided into layers using a metaheuristic approach in the second phase of binary optimization problems, which reduces complexity issues with a low standard deviation of 150.6. The efficacy of this approach is demonstrated by calculating the corona products. The result thus demonstrated that the proposed approach effectively tackles the BO-based optimization problem with an ideal mean value of 12050.

Reference

[1] Chen, S., Dobriban, E. and Lee, J.H., 2020. A group-theoretic framework for data augmentation. *The Journal of Machine Learning Research*, *21*(1), pp.9885-9955.
[2] Hall, M., 2018. *The theory of groups*. Courier Dover Publications.
[3] Neufeld, O., Podolsky, D. and Cohen, O., 2019. Floquet group theory and its application to selection rules in harmonic generation. *Nature communications*, *10*(1), p.405.
[4] Dmitriev, V., 2020. Symmetry Principles and Group Theory in Electromagnetics. *arXiv preprint arXiv:2008.12644*.
[5] Kalaiselvan, S. and Vijayabalaji, S., 2022. Soft Expert Symmetric Group and Its Application in MCDM Problem. *Symmetry*, *14*(12), p.2685.
[6] Monakhov, V.S. and Sokhor, I.L., 2019. Finite groups with abnormal or formational subnormal primary subgroups. *Communications in Algebra*, *47*(10), pp.3941-3949.

[7] Breuillard, E., Guralnick, R. and Larsen, M., 2023. Strongly dense free subgroups of semisimple algebraic groups II. *Journal of Algebra*.

[8] Bansal, J.C. and Deep, K., 2012. A modified binary particle swarm optimization for knapsack problems. *Applied Mathematics and Computation*, *218*(22), pp.11042-11061.

[9] Liu, X.J. and He, Y.C., 2019. Estimation of distribution algorithm based on Lévy flight for solving the set-union knapsack problem. *IEEE Access*, *7*, pp.132217-132227.

[10] Dahmani, I., Ferroum, M., Hifi, M. and Sadeghsa, S., 2020, June. A Hybrid Swarm Optimization-Based Algorithm for the Set-Union Knapsack Problem. In *2020 7th international conference on control, decision and information technologies (codit)* (Vol. 1, pp. 1162-1167). IEEE.

[11] Dahmani, I., Ferroum, M. and Hifi, M., 2021. An iterative rounding strategy-based algorithm for the set-union knapsack problem. *Soft Computing*, *25*(21), pp.13617-13639.

[12] Zhu, W. and Luo, L., 2022. Efficient and Effective Local Search for the Set-Union Knapsack Problem and Budgeted Maximum Coverage Problem. *arXiv preprint arXiv:2207.00749*.

[13] He, Y. and Wang, X., 2021. Group theory-based optimization algorithm for solving knapsack problems. *Knowledge-Based Systems*, *219*, p.104445.

[14] Wilbaut, C., Hanafi, S., Coelho, I.M. and Lucena, A., 2022. The knapsack problem and its variants: formulations and solution methods. In *The Palgrave Handbook of Operations Research* (pp. 105-151). Cham: Springer International Publishing.

[15] Rizk-Allah, R.M. and Hassanien, A.E., 2023. A comprehensive survey on the sine–cosine optimization algorithm. *Artificial Intelligence Review*, *56*(6), pp.4801-4858.

[16] Wang, R. and Zhang, Z., 2021. Set theory-based operator design in evolutionary algorithms for solving knapsack problems. *IEEE Transactions on Evolutionary Computation*, *25*(6), pp.1133-1147.

[17] Li, Y., He, Y., Liu, X., Guo, X. and Li, Z., 2020. A novel discrete whale optimization algorithm for solving knapsack problems. *Applied Intelligence*, *50*, pp.3350-3366.

[18] Lin, G., Guan, J., Li, Z. and Feng, H., 2019. A hybrid binary particle swarm optimization with tabu search for the set-union knapsack problem. *Expert Systems with Applications*, *135*, pp.201-211.

[19] Wei, Z. and Hao, J.K., 2021. Multistart solution-based tabu search for the Set-Union Knapsack Problem. *Applied Soft Computing*, *105*, p.107260.

[20] Feng, Y., Yi, J.H. and Wang, G.G., 2019. Enhanced moth search algorithm for the set-union knapsack problems. *IEEE Access*, *7*, pp.173774-173785.

[21] Wei, Z. and Hao, J.K., 2021. Kernel based tabu search for the Set-union Knapsack Problem. *Expert Systems with Applications*, *165*, p.113802.

[22] Ozsoydan, F.B. and Gölcük, İ., 2023. A reinforcement learning based computational intelligence approach for binary optimization problems: The case of the set-union knapsack problem. *Engineering Applications of Artificial Intelligence*, *118*, p.105688.

[23] García, J., Lemus-Romani, J., Altimiras, F., Crawford, B., Soto, R., Becerra-Rozas, M., Moraga, P., Becerra, A.P., Fritz, A.P., Rubio, J.M. and Astorga, G., 2021. A binary machine learning cuckoo search algorithm improved by a local search operator for the set-union knapsack problem. *Mathematics*, *9*(20), p.2611.

ISSN PRINT 2319 1775 Online 2320 7876

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[24] Ozsoydan, F.B. and Baykasoglu, A., 2019. A swarm intelligence-based algorithm for the set-union knapsack problem. *Future Generation Computer Systems*, *93*, pp.560-569.